A & S 10.1.47 Scattering 24/7/2003

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$$e^{ikz} = \sum_{l=0}^{\infty} (2l+1)\, i^l\, j_l(kR)\, P_l(\cos\theta)$$

$$z = R\cos\theta$$

$$P_l(\cos\theta) = C_{l0}(\theta,\phi) = \sqrt{\frac{4\pi}{2l+1}}\, Y_{l,0}(\theta,\phi)$$

$$e^{ikz} = \sqrt{4\pi} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^l\, j_l(kR)\, Y_{l,0}(\theta,\phi)$$

internal state: $|n j m_j\rangle$

end-over-end: $|l m_l\rangle$

Uncoupled expansion $\psi = \sum_{\substack{n j m_j \\ l m_l}} |n j m_j l m_l\rangle \dfrac{f_{n j m_j l m_l}(R)}{R}$

Flux: $j = \frac{\hbar}{\mu} \mathrm{Im}[\psi^* \nabla \psi]$

$$j_z = \frac{\hbar}{\mu} \mathrm{Im}\left[e^{-ikz} \frac{\partial}{\partial z} e^{ikz}\right] = \frac{\hbar k}{\mu}$$

$$\hat{T} = -\frac{\hbar^2}{2\mu}\frac{\partial^2}{\partial z^2}; \quad \hat{T} e^{ikz} = \frac{\hbar^2 k^2}{2\mu}; \quad k \to k_{n,j}$$

$$E_{tot} = \frac{\hbar^2 k_{n,j}^2}{2\mu} + \varepsilon_{n,j}$$

A & S 10.1.1 $h_l^{(1)}(z) = j_l(z) + i Y_l(z)$ [out]

$h_l^{(2)}(z) = j_l(z) - i Y_l(z)$ [in]

~~$\lim_{z\to\infty} \frac{f_{\ldots}(R)}{R} = \ldots h_l^{(2)}(k_j R)$~~

A & S 10.1.8 $z j_l(z) = \sin(z - \tfrac{1}{2} l\pi)$

10.1.7 $z Y_l(z) = (-1)^{l+1} \cos(z + \tfrac{1}{2} l\pi)$

$$z h_l^{(1)} = \sin(z - \tfrac{1}{2} l\pi) + i(-1)^{l+1}\cos(z + \tfrac{1}{2} l\pi)$$

$$= \frac{e^{i(z-\frac{1}{2}l\pi)} - e^{-i(z-\frac{1}{2}l\pi)}}{2i} + i(-1)^{l+1}\frac{e^{i(z+\frac{1}{2}l\pi)} + e^{-i(z+\frac{1}{2}l\pi)}}{2}$$

$(e^{i\frac{1}{2}\pi} = i)$

$$= e^{iz}\left[\frac{i^{-l}}{2i} + \frac{i(-1)^{l+1} i^l}{2} \cdots \right. \quad l+1$$

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$$i z\, h_l^{(1)}(z) = e^{i(z - l\frac{\pi}{2})}$$

$$i z\, h_l^{(2)}(z) = - e^{-i(z - l\frac{\pi}{2})}$$

Flux normalised waves

Def $$\frac{u_{\underline{n}jl}(R)}{R} = -i h_l^{(2)}(k_{\underline{n}j} R) \left[\frac{\mu k_{\underline{n}j}}{\hbar}\right]^{1/2} \quad \text{in}$$

$$\frac{v_{\underline{n}jl}(R)}{R} = i h_l^{(1)}(k_{\underline{n}j} R) \left[\frac{\mu k_{\underline{n}j}}{\hbar}\right]^{1/2} \quad \text{out}$$

Asymptot. $$\frac{u_{\underline{n}jl}(R)}{R} = e^{-i(k_{\underline{n}j} R - l\frac{\pi}{2})} \left[\frac{\mu}{\hbar k_{\underline{n}j}}\right]^{1/2} \quad \text{in}$$

$$\frac{v_{\underline{n}jl}(R)}{R} = e^{i(k_{\underline{n}j} R - l\frac{\pi}{2})} \left[\frac{\mu}{\hbar k_{\underline{n}j}}\right]^{1/2} \quad \text{out}$$

$$j_l(k_{\underline{n}j} R) = \frac{h_l^{(2)}(k_{\underline{n}j} R) + h_l^{(1)}(k_{\underline{n}j} R)}{2}$$

$$= \frac{i h_l^{(2)}(k_{\underline{n}j} R) + i h_l^{(1)}(k_{\underline{n}j} R)}{2i} \left[\frac{\mu k_{\underline{n}j}}{\hbar}\right]^{1/2} \left[\frac{\hbar}{\mu k_{\underline{n}j}}\right]^{1/2}$$

$$= \frac{-u_{\underline{n}jl}(R) + v_{\underline{n}jl}(R)}{2iR} \sqrt{\frac{\hbar}{\mu k_{\underline{n}j}}}$$

$$e^{i k_{\underline{n}j} \cdot z} |\underline{n} j m_j\rangle = \frac{\sqrt{4\pi}}{2i} \sqrt{\frac{\hbar}{\mu k_{\underline{n}j}}} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^l \left[-\underset{\text{in}}{u_{\underline{n}jl}}(R) + \underset{\text{out}}{v_{\underline{n}jl}}(R)\right]^{(2)} |j m_j l 0\rangle$$

$$= -i \sqrt{\frac{\hbar \pi}{\mu k_{\underline{n}j}}} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^l \left[-u_{\underline{n}jl}(R) + v_{\underline{n}jl}(R)\right] |\underline{n} j m_j l 0\rangle$$

pdc

$$\cancel{\psi^{+}_{\underline{n}jm_j} = -i\sqrt{\frac{4\pi}{\mu k_{\underline{n},j}}} \sum_{l=0}^{\infty} \sum_{m_l=0}^{\infty} \sqrt{2l+1}}$$

Solve uncoupled eq with b.c.

$$\psi^{\underline{n}jlm_l}(R) \simeq -u_{\underbrace{\scriptstyle \underline{n}jlm_l}_{|\underline{n}jlm_l\rangle}}(R) + \sum_{\substack{\underline{n}'j'l'm_l' \\ m_j'}} v_{\underbrace{\scriptstyle \underline{n}'j'l'}_{|\underline{n}'j'l'm_l'\rangle}}(R)\, S_{\underline{n}'j'l'm_l';\,\underline{n}jlm_l}$$

$$\psi^{+}_{\underline{n}jm_j} = -i\sqrt{\frac{4\pi}{\mu k_{\underline{n},j}}} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^{l}\, \psi^{\underline{n}jl0}_{m_j}$$

check ingoing part ... OK

Outgoing:

$$-i\sqrt{\frac{4\pi}{\mu k_{\underline{n},j}}} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^{l} \sum_{\substack{\underline{n}'j'l'm_l \\ m_j'}} |\underline{n}'j'l'm_l'\rangle\, v_{\underline{n}'j'l'}(R)\, S_{\underline{n}'j'l'm_j';\,\underline{n}jm_j l0}$$

asympt $v_{\underline{n}',j'l'}(R) \sim e^{ik_{\underline{n}',j'}R}\, e^{-il'\frac{\pi}{2}} \left[\frac{\mu}{\hbar k_{\underline{n},j'}}\right]^{1/2}$

b.c.

~~outgoing in channel m_j~~

$$e^{ik_{\underline{n}j}z}|\underline{n}jm_j\rangle + \sum_{\substack{\underline{n}'j' \\ m}} |\underline{n}'j'm_j'\rangle \frac{e^{ik_{\underline{n}'j'}R}}{R} \times f_{\underline{n}'j'm_j'}(\theta,\phi)$$

$$f_{\underline{n}'j'm_j'}(\theta,\phi) = -i\sqrt{\frac{4\pi}{\mu k_{\underline{n},j}}} \sum_{l=0}^{\infty} \sqrt{2l+1}\, i^{l} \sum_{l'} e^{-il'\frac{\pi}{2}} \left[\frac{\mu}{\hbar k_{\underline{n}'j'}}\right]^{1/2}$$

$$\sum_{m_l'} \left(S_{\underline{n}'j'l'm_l';\,\underline{n}jl0\,m_j'\,m_j} - \delta_{\underline{n}'\underline{n}}\delta_{j'j}\delta_{l'l}\delta_{m_l'0} \right) Y_{l'm_l'}(\theta,\phi)$$

$$e^{-il'\frac{\pi}{2}} = i^{-l'}$$

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$$f_{n' j' m_j'}(\theta,\phi) = -i \frac{\sqrt{\pi}}{\sqrt{k_{nj}\, k_{n'j'}}} \sum_{l l' m_l'} \sqrt{2l+1}\; i^{\,l-l'}\; Y_{l' m_l'}(\theta,\phi) \underbrace{\left[S_{n' j' l' m_l',\, n j l 0} - \delta_{n n'} \delta_{j j'} \delta_{l l'} \delta_{m_l' 0} \right]}_{\delta_{m_j' m_j}}$$

radial flux $\frac{\hbar}{\mu} k_{n' j'}$

$$\frac{d\sigma}{d\Omega} = \frac{j_{out}}{j_{in}} = \frac{k_{n' j'}}{k_{n j}} \sum_{m_j'} \left| f_{n' j' m_j'}(\theta,\phi) \right|^2 \qquad j\, m_j \to j'\, m_j'$$

$$= \frac{\pi}{|k_{nj}|^2} \sum_{m_j'} \left| \sum_{l l' m_l'} \sqrt{2l+1}\; i^{\,l-l'}\; Y_{l' m_l'}(\theta,\phi)\; T_{n' j' l' m_l',\, n j l 0} \right|^2$$

$$S_{n' j' m_j' l' m_l',\, n j m_j l m_l} = \sum_{JM} \langle j' m_j' l' m_l' | JM \rangle\, S^J_{n' j' l',\, n j l}\, \langle JM | j m_j l m_l \rangle \qquad m_l = 0$$

$$\sum_{JM} \langle j' m_j' l' m_l' | JM \rangle\, \delta_{n n'} \delta_{j j'} \delta_{l l'}\, \langle JM | j m_j l 0 \rangle$$

$$= \delta_{n n'} \delta_{j j'} \delta_{l l'} \underbrace{\sum_{JM} \langle j' m_j' l m_l' | JM \rangle \underbrace{\langle JM | j m_j l 0 \rangle}_{\delta_{M, m_j}}}_{\delta_{m_l' m_l}\; \delta_{m_j' m_j}}$$

$$\sigma_{n' j' m_j' \leftarrow n j m_j}(\theta,\phi) = \frac{\pi}{|k_{nj}|^2} \left| \sum_{l l' m_l' J \underset{m_j}{M}} \sqrt{2l+1}\; i^{\,l-l'}\; Y_{l' m_l'}(\theta,\phi)\, \langle j' m_j' l' m_l' | JM \rangle \left[S^J_{n' j' l',\, n j l} - \delta_{n n'} \delta_{j j'} \delta_{l l'} \right] \langle j m_j l 0 | J \underset{m_j}{M} \rangle \right|^2$$

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