derivation of the time dependent normalisation constat

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$$\alpha_t = \frac{\alpha_0 \mu}{\mu + i\hbar(t-t_0)\alpha_0}$$

$$= \frac{\alpha_0 \mu^2}{\mu^2 + \hbar^2\alpha_0^2(t-t_0)^2} - \frac{i\hbar(t-t_0)\alpha_0^2 \mu}{\mu + \hbar^2\alpha_0^2(t-t_0)^2}$$

when normalizing the wave function

$$\langle \Psi(x,t) | \Psi(x,t) \rangle = \underbrace{e^{\alpha_t^* + \alpha_t} \; e^{c_t + c_t^*}} \int_{-\infty}^{\infty} dx \; e^{-(x-vt)^2}$$

↳ we need to have that this part stays constant, i.e. has no time dependence.

$$a + a^* = 2\,\mathrm{Re}(a)$$

$$2\,\mathrm{Re}(\alpha_t) = \frac{2\alpha_0 \mu^2}{\mu^2 + \hbar^2\alpha_0^2(t-t_0)^2}$$

which means that

$$2\,\mathrm{Re}(c_t) = -2\,\mathrm{Re}(\alpha_t) = -\frac{2\alpha_0 \mu^2}{\mu^2 + \hbar^2\alpha_0^2(t-t_0)^2}$$

but also can have an arbitrary term summed with it

$$2\,\mathrm{Re}(c_t) = -\frac{2\alpha_0 \mu^2}{\mu^2 + \hbar^2\alpha_0^2(t-t_0)^2} + C$$

so that

$$\langle \Psi(x,t) | \Psi(x,t) \rangle = e^{2\mathrm{Re}(\alpha_t)} \; e^{-2\mathrm{Re}(\alpha_t)} \; e^{C} \int_{-\infty}^{\infty} dx \; e^{-(x-vt)^2}$$

Say $e^{c} = |A|$, and compute

$$\int_{-\infty}^{\infty} dx \; e^{-(x-vt)} = \sqrt{\pi}$$

$$\Rightarrow \quad |A| = \frac{1}{\sqrt{\pi}}$$

$$c = -\tfrac{1}{2}\ln(\pi)$$

$\Rightarrow$ Since C_t is a normalisation factor and we only need its real part we can safely say that it is real. We have found it to be

$$C_t = -\frac{\alpha_0 \mu^2}{\mu^2 + \hbar^2 \alpha_0^2 (t-t_0)^2} - \frac{1}{4}\ln(\pi)$$