QTC, NWI-MOL112, exercises week 4

Gerrit C. Groenenboom, Feb 26, 2020

Question 1: Questions chapter 7

For Jacobi vectors $\boldsymbol{r}$ and $\boldsymbol{R}$ the corresponding angular momentum operators are defined by

$$\hat{\boldsymbol{j}} = \boldsymbol{r} \times \hat{\boldsymbol{p}}_r \tag{1}$$

$$\hat{\boldsymbol{l}} = \boldsymbol{R} \times \hat{\boldsymbol{p}}_R \tag{2}$$

where $\hat{\boldsymbol{p}}_r$ and $\hat{\boldsymbol{p}}_R$ are the momentum operators for $\boldsymbol{r}$ and $\boldsymbol{R}$, respectively. The total angular momentum operator is defined by

$$\hat{\boldsymbol{J}} = \hat{\boldsymbol{j}} + \hat{\boldsymbol{l}}. \tag{3}$$

1a. Derive the following commutation relations, for $i = x, y, z$,

$$[\hat{\boldsymbol{J}}_i, r^2] = 0 \tag{4}$$

$$[\hat{\boldsymbol{J}}_i, R^2] = 0 \tag{5}$$

$$[\hat{\boldsymbol{J}}_i, \boldsymbol{r} \cdot \boldsymbol{R}] = 0 \tag{6}$$

1b. For two Hermitian matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ that commute, $[\boldsymbol{A}, \boldsymbol{B}] = \boldsymbol{0}$, show that

$$e^{\boldsymbol{A}+\boldsymbol{B}} = e^{\boldsymbol{A}} e^{\boldsymbol{B}}$$

1c. Show that the previous result also holds if the matrices are not Hermitian (but still commute).

1d. Use the method described in chapter 7.5 to find the coupled angular momentum state $|(jl)JM\rangle$ with $j = 2,\ l = 3,\ J = 5,\ M = 4$.

1e. Use the method described in chapter 7.5 to find the coupled angular momentum state $|(jl)JM\rangle$ with $j = 2,\ l = 3,\ J = 4,\ M = 4$.