

Mechanics 2, exercises week 5, NWI-MOL160

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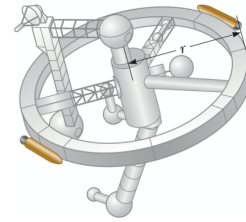
Question 1: Manoeuvring spacecraft

A spacecraft is in empty space. It carries on board a gyroscope with a moment of inertia of $I_g = 20.0 \text{ kg m}^2$ about the axis of the gyroscope. The moment of inertia of the spacecraft around the same axis is $I_s = 5.00 \cdot 10^5 \text{ kg m}^2$. Neither the spacecraft nor the gyroscope is originally rotating. The gyroscope can be powered up in a negligible period of time to an angular speed of 100 rad/s .

- 1a. If the orientation of the spacecraft is to be changed by 30.0° , for what time interval should the gyroscope be operated?

Question 2: Rotating space station

A space station shaped like a giant wheel has a radius of $r = 100 \text{ m}$ and a moment of inertia of $5.00 \cdot 10^8 \text{ kg m}^2$. A crew of 150 people of average mass 65.0 kg is living on the rim. The station's rotation causes the crew to experience an apparent free-fall acceleration of g .



- 2a. What is the required angular velocity (in rev/min) of the station in order to maintain an Earth-like gravitation?
- 2b. One evening, 100 average people move to the center of the station for a union meeting. What is the effect on the “artificial gravity” in the stations rim?

Question 3: Avoiding space debris

You are in a spacecraft in a circular orbit going around Earth at a distance $r_i = 10,000 \text{ km}$ from the center of the Earth.

- 3a. How long does it take to go around Earth in this orbit?

You get a warning about space debris in your orbit. To climb to a higher orbit you point down the rocket engine of your spacecraft and you briefly switch it on to gain a vertical speed $v_i = 1000 \text{ m/s}$. After switching off your engine, according to Kepler, you must be in an elliptic orbit.

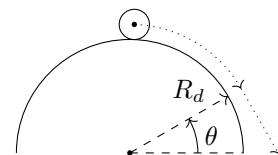
- 3b. What are the aphelion r_a and perihelion r_p of your new orbit? (*Hint: you will need the effective potential and the radial kinetic energy.*)

After solving this question you realize you are still somewhat at risk from debris in the orbit with radius r_i (why?), so you start planning to fire your engine again once you reach a distance $r_f = r_a$ from the center of Earth. This time you will fire the rocket horizontally, i.e., you will accelerate in the direction your spacecraft is moving.

- 3c. By what Δv should you increase your speed to end up in a circular orbit with radius r_f ?
Hint: what happens with the angular momentum of the spacecraft?

Question 4: Rolling a coin off of a dome

You are sitting at the top of a spherical dome with radius R_d and roll a coin down the dome. Assume that the radius of the coin $r \ll R_d$. Initially, the latitude of the coin is $\theta_i = 90^\circ$, and it starts rolling with negligible speed. At some latitude $\theta = \theta_f$ the coin will fly off the dome.



- 4a. What is θ_f if you simplify the problem by assuming the coin slides frictionless down the Dome?
- 4b. What is θ_f be if the coin rolls without slipping?