

Mechanics 2, exercises week 4, NWI-MOL160

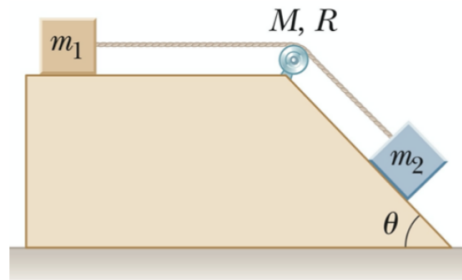
Prof. Gerrit C. Groenenboom, 23-Sep-2026

Question 1: Rolling vs. Sliding

A solid sphere of mass m and radius R is released from height h at the top of an incline making an angle θ with the horizontal. The moment of inertia of a solid sphere is $\frac{2}{5}mR^2$

- 1a. Calculate the speed of the sphere if it rolls without slipping.
- 1b. Calculate the speed for the case of the sphere sliding without friction.
- 1c. Compute the ratio of the times it takes to reach the bottom of the incline for cases (a) and (b).

Question 2: Sliding masses



A block of mass $m_1 = 2$ kg and a block of mass $m_2 = 6$ kg are connected by a massless string over a pulley in the shape of a solid disk (with radius $R = 0.25$ m and mass $M = 10.0$ kg). The fixed ramp (as shown) makes an angle of $\theta = 30^\circ$ deg

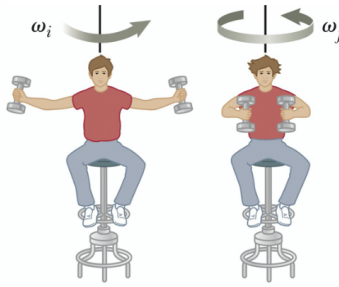
- 2a. Using as coordinate the x -coordinate of the mass m_1 , write down the kinetic energy as a function of the velocity $\dot{x}$
- 2b. Find the momentum conjugate to coordinate x
- 2c. Write down the Hamiltonian of the system
- 2d. Write down Hamilton's equation of motion for position and momentum
- 2e. Assuming the system is initially, at time $t = 0$ at rest, solve the equations of motion.
- 2f. Find the tension in the string *hint: the acceleration of the mass m_1 is due to the tension in the string*

Question 3: Angular momentum of the Earth

Modelling Earth as a perfect uniform sphere, with mass $M_E = 5.97 \times 10^{24}$ kg, radius $R_E = 6.37 \times 10^6$ m, and distance to the sun, $d_E = 1.496 \times 10^{11}$ m, compute:

- 3a. The angular momentum of the earth due to the spinning motion about its axis
- 3b. The angular momentum of the Earth due to its orbital motion about the sun
- 3c. Explain why the answer in (b) is larger than the answer in (a), even though it takes significantly longer for earth to orbit the sun than rotate on its axis.

Question 4: Rotating chair



A student sits on a freely rotating stool holding two dumbbells, each of mass 3 kg. When his arms are extended horizontally, the dumbbells are 1.0 m from the axis of rotation and the student rotates with an angular speed of 0.75 rad/s. The moment of inertia of the student plus stool is 3.00 kg m^2 and is assumed to be constant. The student pulls the dumbbells inward horizontally to a position 0.30 m from the rotation axis.

- 4a. What is the new angular speed of the student?
- 4b. What is the kinetic energy of the rotating system before and after he pulls the dumbbells inwards?

Question 5: Astronaut throwing object from spacecraft

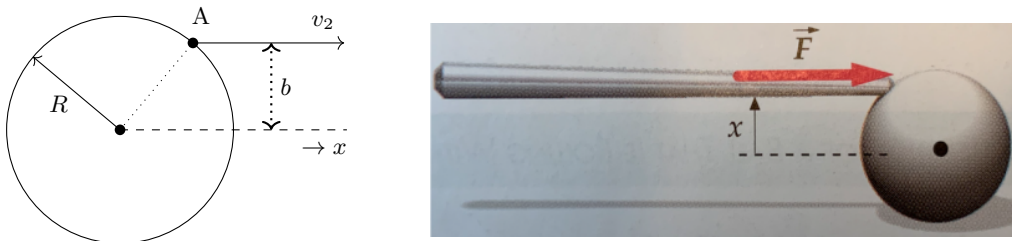


Figure 1. *Left panel:* Astronaut (A) on spherical spacecraft. *Right panel:* horizontal cue hitting billiard ball.

An astronaut (A) sits on the outside of a spacecraft and throws away an object. We model the spacecraft as a solid, perfect sphere, with mass m_1 and radius R and the object as a point particle with mass m_2 . The mass of the astronaut is negligible. The object is thrown in the x direction of a frame with the initial center of the spacecraft as origin (see Figure). The initial y -coordinate of the astronaut is b . The velocity of the object after it was thrown is v_2 . Assume that initially the spacecraft is not rotating.

- 5a. Use conservation of linear momentum to compute the velocity v_1 of the CoM of the spacecraft after the object was thrown.
- 5b. Use conservation of angular momentum to compute the angular velocity of the spacecraft after the object was thrown.
- 5c. At what height x (see figure) should a horizontal cue hit a billiard ball so it will roll without slipping?