Mechanics 2, exercises week 1, NWI-MOL160

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In these questions, start by asking yourself:

- Do I need to split this problem in separate steps?
- What are the conserved quantities? Is energy conserved? Is momentum conserved?
- In what frame should I solve this problem? Should I use the lab-frame? Should I use a center-of-mass frame?
- Can I write down the Hamiltonian for this problem?

Also, define symbols for the quantities that are given, putting back the numbers only in the last step of the calculation.

Question 1: Reduced mass of two particles

Two particles with masses m_1 and m_2 are moving on a straight line. Their positions in a center-of-mass frame are x_1 and x_2. The kinetic energy expressed in momenta $p_1 = m_1\dot{x}_1$ and $p_2 = m_2\dot{x}_2$ is

$$T_a = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2}. \tag{1}$$

The kinetic energy can also be written as

$$T_b = \frac{p_r^2}{2\mu}, \tag{2}$$

where the relative momentum $p_r = \mu\dot{r}$ and $r = x_2 - x_1$ the distance between the particles.

1a. We have the relation $p_1 = -p_2$, why is that?

1b. Show that in order to have $T_a = T_b$, the reduced mass must be given by

$$\mu = \frac{m_1 m_2}{m_1 + m_2}. \tag{3}$$

Question 2: A skateboard workout

A 50 kg skateboarder stands on a 3 kg skateboard, holding two 5 kg weights in her hands. Starting from rest, she throws the weights horizontally, one at a time. The speed of each weight is 8 m/s relative to her and the board after it is thrown. Assume the board rolls without friction and neglect air resistance, etc.

2a. How fast is she going after throwing the first weight?

2b. How fast is she going after throwing the second weight?

2c. Throwing the weights costs energy. How much energy did throwing the first weight cost (at least), and how much energy did throwing the second weight cost (at least)? The "at least" was added, since throwing something will also warm up muscles, etc., and that is not considered here.

2d. Compute the total kinetic energy of skateboarder, the skateboard, and the two weights, after the second weight was thrown. Does this match the sum of the energies in the previous question?

2e. How fast would she be if she had thrown both weights at the same time, with the weights again traveling a 8 m/s relative to her?

Question 3: Ball in the air

A ball of mass m is thrown straight up into the air with an initial speed v_i.

3a. What is the momentum of the ball at its maximum height?

3b. What is the momentum of the ball halfway to its maximum height?

Question 4: Measuring momentum

A pendulum consists of a ball with a mass of 2 kg suspended on a string with a length of 75 cm. Initially it is a rest. The ball, made of wood, is hit by a bullet that was moving horizontally, and the bullet gets stuck in the wood. As a result, the pendulum starts swinging, and the deflection angle reaches 45° (maximum).

4a. What was the linear momentum of the bullet, assuming that the bullet is much lighter than 2 kg?

4b. If the mass of the bullet was 10 g, then what was its initial velocity?

Question 5: Velocity map imaging in 1D

In a lab experiment, a beam of OH radicals moving with a speed of 300 m/s is pointed at another beam consisting of Xe atoms moving in the opposite direction, also with 300 m/s. Some of the OH radicals hit a single Xe atom and bounce right back. In an actual experiment, collisions will of course not happen in 1D, but it is actually possible to measure the velocity of OH radicals that happen to move in a specific direction after a collision [1]. The mass of the OH radical is 17 u and the mass of a Xe atom is 131 u, where the atomic mass unit u = 1.66×10^{-27} kg.

5a. For an elastic collision of OH+Xe, what would be the velocity of OH after the collision?

Due to the collision, the OH radical may actually start rotating and/or vibrating and some of the translational kinetic energy is converted into internal energy of OH. As a result, the OH radicals will move a bit slower after the collision than in the case of elastic collision.

5b. If the OH radicals move 10 m/s slower than in the case of an elastic collision, how much rotation/vibration energy did the OH radical gain in the collision?

References

[1] **Near-threshold inelastic collisions using moleculear beams with a tunable velocity**, J. J. Gilijamse, S. Hoekstra, S. Y. T. van de Meerakker, G. C. Groenenboom, and G. Meijer, Science **313**, 1617 (2006).