ii. Introduction to BEC (continues...)
Reaching BEC

✓ Typical P-T phase diagram

\[ P = \frac{2E}{3V} = \zeta(5/2) \left( \frac{m}{2\pi\hbar^2} \right)^{3/2} (kT)^{5/2} \]

✓ BEC can be realised only in conditions of metastability

✓ Low density: three body recombination rate \ll\ two body scattering rate

\[ n \sim 10^{13} - 10^{15}\text{atom/cm}^3 \]

\[ T \sim 500nK - \mu K \]

✓ Kinetic vs. chemical equilibrium (two body collisions ensure thermalization, but the gas has a finite lifetime - few sec.)

very sophisticated cooling and trapping techniques
Magnetic Trapping

✓ Atoms in an inhomogeneous field experience a spatially-varying potential -- adiabatic approximation $B>0$)
  • high-field-seeking states ($\mu_i > 0$)
  • low-field-seeking states ($\mu_i < 0$) -- no maxima of the magnetic field in vacuum

$$E_i \simeq C_i - \mu_i B$$

$F = 2$
$F = 1$

$\text{Li}^7, \text{Na}^{23}, \text{Rb}^{87}, \ldots$

$m_F = \pm 2$
$\pm 1$
$0$
$-1$

$\pm 2$
$\pm 1$
$0$
$-1$
Magnetic Trapping

- high-field-seeking states ($\mu_i > 0$)
- low-field-seeking states ($\mu_i < 0$)

\[ E_i \simeq C_i - \mu_i B \]

\[ ^{87}\text{Rb} \]
Magnetic Trapping

✓ Linear Quadrupole Trap

\[ \mathbf{B} = B' \begin{pmatrix} x \\ y \\ -2z \end{pmatrix} \]

\[ U_{\text{pot}} \propto |\mathbf{B}| \]

leaking trap (Majorana flops)

✓ TOP Trap (JILA)

\[ \mathbf{B} = \begin{pmatrix} B'x + B_0 \cos(\omega t) \\ B'y + B_0 \sin(\omega t) \\ -2B'z \end{pmatrix} \]

\[ U_{\text{pot}} \propto |\mathbf{B}| \]

[W. Petrich et al. PRL 74, 3352 (1995)]
Magnetic Trapping

✓ Linear Quadrupole Trap

\[ B = B' \begin{pmatrix} x \\ y \\ -2z \end{pmatrix} \]

leaking trap (Majorana flops)

✓ Optical plug (MIT)

A repelling (blue detuned) laser field is applied in the vicinity of the node

[K. B. Davis et al. PRL 75, 3969 (1995)]
# Evaporative Cooling

<table>
<thead>
<tr>
<th>Method</th>
<th>Temperature</th>
<th>Density</th>
<th>Phase Space Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oven</td>
<td>$T \simeq 500 \text{K}$</td>
<td>$n \simeq 10^{11} \text{cm}^{-3}$</td>
<td>$\rho \simeq 10^{-13}$</td>
</tr>
<tr>
<td>Laser Cooling</td>
<td>$T \simeq 50 \mu\text{K}$</td>
<td>$n \simeq 10^{14} \text{cm}^{-3}$</td>
<td>$\rho \simeq 10^{-6}$</td>
</tr>
<tr>
<td>Evaporative Cooling</td>
<td>$T \simeq 500 \text{nK}$</td>
<td>$n \simeq 3 \times 10^{14} \text{cm}^{-3}$</td>
<td>$\rho \simeq 1$</td>
</tr>
</tbody>
</table>

Phase space density

$$\rho = n \lambda_T^3 = n \left(\frac{2\pi \hbar^2}{mkT}\right)^{3/2}$$

($\rho_c \simeq 2.6$)

Thermal distribution atoms in the trap

Remove the tail of hot atoms with a rf flip

$$\epsilon > \eta kT$$

Maxwell-Boltzmann
Imaging Techniques

- Resonant absorption imaging (momentum) density distribution of the expanded cloud
- Phase contrast imaging (spatial) density distribution of the trapped cloud
Observing a BEC in $^{87}\text{Rb}$

[M. H. Anderson et al. Science 269, 198 (1995)]

oven $T \simeq 300\text{K}$
laser cooling $T \simeq 90\mu\text{K}$

$n \simeq 2 \times 10^{10} \text{ cm}^{-3}$

$|F = 2, m_F = 2\rangle$
evaporative cooling in TOP trap $T_c \simeq 170\text{nK}$

$n \simeq 2.6 \times 10^{12} \text{ cm}^{-3}$

evolution & probe
Observing a BEC in $^{23}\text{Na}$

$|F = 1, m_F = -1\rangle$  

evaporative cooling in optical plug trap  

$T_c \approx 2.0 \mu\text{K}$  

$n \approx 1.5 \times 10^{14} \text{ cm}^{-3}$

✓ Bimodal distribution  
✓ Non-isotropic velocity distribution

[K. B. Davis et al. PRL 75, 3969 (1995)]
Non-Destructive Observation in $^{23}\text{Na}$

[M. R. Andrews et al. Science 273, 84 (1996)]

✓ Direct (space) imaging of the condensate inside the magnetic trap by dispersive light scattering
Interference Between Two Bose Condensates

\[ \lambda = \frac{ht}{md} \]

\[ t = 40 \text{ms} \]
\[ d \approx 40 \mu \text{m} \]
\[ \lambda \approx 20 \mu \text{m} \]
Interactions between Atoms

✓ Alkali atomic gases are very dilute

\[ n^{-1/3} \sim 100\text{nm} \quad a \sim 100a_0 \sim 5\text{nm} \]

✓ Two-body scattering mechanisms

• ensure thermalization (kinetic equilibrium, evaporative cooling, ...)
• determine equilibrium shape and dynamics of the condensate
• can be tuned: Feshbach resonances
Interactions between Atoms

✓ Interatomic potential

\[ \hat{H} = \frac{d_1 \cdot d_2 - 3(d_1 \cdot \hat{r})(d_2 \cdot \hat{r})}{4\pi \epsilon_0 r^3} \]

large distance dipole-dipole interaction

\[ U_{\text{VdW}} = -\frac{C_6}{r^6} \]

Van der Waals attraction

Potential energy (arb. units)

Interatomic distance (arb. units)
Interactions between Atoms

✓ At small distances, the interaction is predominantly determined by the valence electrons ($S=0$ singlet, $S=1$ triplet)

\[ \hat{H}_{\text{int}} = \frac{U_s + 3U_t}{4} + (U_t - U_s) \hat{S}_1 \cdot \hat{S}_2 \]

✓ N.B. $\hat{S}_1 \cdot \hat{S}_2$ can flip the two atoms spin($\uparrow\downarrow\rightarrow\downarrow\uparrow$) and therefore change the initial hyperfine states

multi-channel problem
Basic Scattering Theory

(see blackboard)

✓ Evaluate the s-wave scattering length

\[
\left( \frac{\hat{p}^2}{2m} + V(\hat{r}) \right) \psi_k(r) = \frac{\hbar^2 k^2}{2m_r} \psi_k(r) \quad \psi_k(r) = \sum_{l=0}^{\infty} A_l P_l(\cos \theta) \frac{u_{kl}(r)}{k r}
\]

\[
\frac{d^2 u_{kl}(r)}{dr^2} + \left[ k^2 - \frac{l(l + 1)}{r^2} - \frac{2m_r}{\hbar^2} V(r) \right] u_{kl}(r) = 0
\]

✓ Low energy asymptotics

\[ u_{00}(r) \propto r - a \]

s-wave scattering length

✓ Scattering cross section

\[
\sigma = 8\pi a^2 \quad \text{for bosons}
\]

\[
\sigma = 0 \quad \text{for fermions}
\]
	no s-wave scattering for identical fermions at low energies
$u(r) = \begin{cases} C_1(r - a) \\ C_2 \sin(k_0 r) \end{cases}$

$k_0 = \sqrt{2m_r V_0}$

\[
\frac{a}{b} = 1 - \frac{\tan(k_0 b)}{k_0 b}
\]
Square Potential Well

i. \( k_0b < \pi/2 \) \( \Rightarrow a < 0 \) (effective attractive interaction!) \( V_0 \) is too small to have a bound state

ii. Increasing \( V_0 \) (fixing \( b \)) the scattering length goes to \( \infty \) every time the potential can hold a new bound state (general relation)

iii. \( a < 0 \) (attractive interaction) & \( a > 0 \) (repulsive interaction)

iv. At \( k_0b \gg 1 \), more likely \( a > 0 \)

\[
\frac{a}{b} = 1 - \frac{\tan(k_0b)}{k_0b}
\]

\[
k_0 = \sqrt{2m_rV_0}
\]
More Realistic Potential

[G. F. Gribakin et al. PRA 48, 546 (1993)]

Truncated $r^{-6}$ potential

$$V(r) = \begin{cases} +\infty & r < r_c \\ -C_6/r^6 & r \geq r_c \end{cases}$$

![Graph of the truncated $r^{-6}$ potential](image)

- Relative variation of the $C_6$ coefficient
- a (Angstroms)