$$\sigma_{jm_j;j'm_j'}(\underline{k}) = \frac{4\pi^2\omega}{c} \langle \hat{k} E^- jm_j | \underline{e}\cdot\underline{d} | \hat{f}_{K_iQ_i}(J_i,J_i') | \underline{e}^* \underline{d} | \hat{k} E^- j'm_j' \rangle$$

$$\sigma_{KQ}(j,j';\underline{k}) = \frac{4\pi^2\omega}{c}\sum_{m_j m_j'} \langle \hat{k} E^- jm_j | \underline{e}\cdot\underline{d} | \hat{f}_{K_iQ_i}(J_i,J_i') | \underline{e}^* \underline{d} | \hat{k} E^- j'm_j' \rangle [K]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m}_j & Q & m_j' \end{pmatrix}$$

$$= \frac{4\pi^2\omega}{c} \sum_{kq} [\hat{e}\times\hat{e}^*]^{(k)}_{q_*} \sum_{\mu\mu'} \sum_{m_j m_j'} \langle \hat{k} E^- jm_j | d_\mu | \hat{f}_{K_iQ_i}(J_i,J_i') | d_{\mu'} | \hat{k} E^- j'm_j' \rangle$$

$$[K]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m}_j & Q & m_j' \end{pmatrix} \langle 1\mu\, 1\mu' | k q_z' \rangle \langle k q_* \, k q_z' | 00 \rangle [k]^{1/2}$$

$$| \hat{k} E^- j m_j \rangle = \frac{-i}{\sqrt{2\pi}} \sum_{l m_l J M \Omega} | j\Omega J M \rangle\, u^{J j}_{\Omega l} \langle j m_j\, l m_l | J M \rangle Y^*_{l m_l}(\hat{k})$$

$$u^{Jj}_{\Omega l} = \langle j\Omega\, l 0 | J\Omega \rangle \frac{[l]^{1/2}}{[J]^{1/2}}$$

$$\hat{f}_{K_iQ_i}(J_i,J_i') = \sum_{M_iM_i'} | J_iM_i \rangle \langle J_i'M_i' | \, [K_i]^{1/2} \begin{pmatrix} J_i & K_i & J_i' \\ \bar{M}_i & Q_i & M_i' \end{pmatrix}$$

$$E_{kq} = [\hat{e}\times\hat{e}^*]^{(k)}_q = \sum_{mm'} e_m (e^*)_{m'} \langle 1m\, 1m' | kq \rangle$$

$$= [k]^{1/2} \sum_{mm'} (-1)^m e_m e^*_{m'} \begin{pmatrix} 1 & 1 & k \\ m & -m' & -q \end{pmatrix}$$

$$\sigma_{KQ}(j,j';k) = \frac{4\pi^2\omega}{c} \sum_{\substack{kq \\ l m_l \\ l' m_l'}} \sum_{\substack{\mu\mu' MM'\Omega\Omega' \\ m_j m_j' JJ'}} E_{kq} \langle j\Omega J M | d_\mu | \hat{f}_{K_iQ_i}(J_i,J_i') | d_{\mu'} | j'\Omega' J'M' \rangle$$

1 2 3 4 5

$$\left(\frac{-1}{2\pi}\right)[K]^{1/2}[k]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m}_j & Q & m_j' \end{pmatrix} \langle 1\mu\, 1\mu' | kq' \rangle \langle kq\, kq' | 00 \rangle$$

6 7 9 10

$$u^{Jj}_{\Omega l} \langle j m_j\, l m_l | J M \rangle\, v^{J'j'}_{\Omega' l'} \langle j' m_j'\, l' m_l' | J'M' \rangle\; Y_{l m_l}(\hat{k}) Y^*_{l' m_l'}(\hat{k})$$

$$Y_{lm_l}(\hat{k}) Y^*_{l'm_l'}(\hat{k}) = \frac{[l]^{1/2}[l']^{1/2}}{4\pi} C_{lm}(\hat{k})\, C^*_{l'm_l'}(\hat{k}) (-1)^{m_l'}$$

$$= \sum_{LM_L} \frac{[l]^{1/2}[l']^{1/2}}{4\pi} \sum_{LM_L} (-1)^{m_l'} \langle l m_l\, l' -m_l' | L M_L \rangle \langle l0\, l'0 | L0 \rangle C_{LM_L}(\hat{k})$$

8 11

W.E.

$$\langle J_i M_i | d_\mu | j\Omega J M\rangle = [1]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & \mu & M \end{pmatrix} \langle J_i \| d \| j\Omega J\rangle$$

$$\langle j\Omega J M | d_\mu | J_i M_i\rangle = \langle J_i M_i | d_\mu^* | j\Omega J M\rangle^*$$

$[d_\mu^* = (-1)^\mu d_{-\mu}]$

$$= (-1)^\mu [1]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & -\mu & M \end{pmatrix} \langle J_i \| d \| j\Omega J\rangle^*$$

$$\sum_{M_i M_i'} \underbrace{\langle j\Omega J M | d_\mu | J_i M_i\rangle} \underbrace{\langle J_i' M_i' | d_{\mu'} | j'\Omega' J' M'\rangle} [K_i]^{1/2} \begin{pmatrix} J_i & K_i & J_i' \\ \bar{M}_i & Q_i & M_i' \end{pmatrix}$$

$$= 3 \langle J_i \| d \| j\Omega J\rangle^* \langle J_i' \| d \| j'\Omega' J'\rangle$$

$$\sum_{M M' M_i} (-1)^\mu \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & -\mu & M \end{pmatrix} \begin{pmatrix} J_i' & 1 & J' \\ \bar{M}_i' & \mu' & M' \end{pmatrix} [K_i]^{1/2} \begin{pmatrix} J_i & K_i & J_i' \\ \bar{M}_i & Q_i & M_i' \end{pmatrix}$$

$$|f\rangle = |\vec{k}\, \epsilon^{-}\, j m_j (R)\rangle = \frac{-i}{\sqrt{2\pi}} \sum_{l m_l J M \Omega} |j\Omega J M(R)\rangle\, \mathcal{U}^{Jj}_{\Omega l}\, \langle j m_j\, l m_l | J M\rangle\, Y^{*}_{l m_l}(\hat{k})$$

$$\mathcal{U}^{(Jj)}_{\Omega l} = \langle j \Omega\, l 0 | J \Omega\rangle \frac{[l J]^{1/2}}{[J]^{1/2}}$$

$$\hat{\rho}^{(i)}_{K_i Q_i}(J_i', J_i) = \sum_{M M_i'} |J_i' M_i'\rangle \langle J_i M_i| \quad [K_i]^{1/2} \begin{pmatrix} J_i' & K & J_i \\ \bar{M}_i' & Q & M_i \end{pmatrix}$$

$$\sigma_{KQ}(j'; j; k) = \sum_{m'm} \sigma_{j'm'; jm} \quad [K]^{1/2} \begin{pmatrix} j' & K & j \\ \bar{m}' & Q & m \end{pmatrix}$$

$\sigma_{j'm'; jm}$

$$\sum_{m_1 m_2} c \langle f' | \hat{\mu}_{m_1} | \hat{\rho}_{KQ} | \hat{\mu}_{m_2} | f^{*}\rangle \langle 1 m_1\, 1 m_2 | k q\rangle$$

$$= c \frac{-1}{2\pi} \sum_{m_1 m_2} \sum_{\substack{l m_l J M \Omega \\ l' m_l' J' M' \Omega'}} \langle j \Omega' J' M'(R) | \hat{\mu}_{m_1} | J_i' M_i'\rangle \langle J_i M_i | \hat{\mu}_{m_2} | j \Omega J M(R)\rangle$$

$$[K_i]^{1/2} \begin{pmatrix} J_i' & K_i & J_i \\ \bar{M}_i' & Q_i & M_i \end{pmatrix} \quad \langle 1 m_1\, 1 m_2 | k q\rangle \qquad [K]^{1/2} \begin{pmatrix} j' & K & j \\ \bar{m}' & Q & m \end{pmatrix}$$

$$\mathcal{U}^{J'j'}_{\Omega' l'} \langle j' m_j'\, l' m_l' | J' M'\rangle \quad \mathcal{U}^{Jj}_{\Omega l} \langle j m_j\, l m_l | J M\rangle$$

$$Y_{l m_l}(\hat{k})\, Y^{*}_{l' m_l'}(\hat{k})$$

W.E. $\langle J_i M_i | \hat{\mu}_{m_2} | j \Omega J M(R)\rangle = \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & m_2 & M \end{pmatrix} \langle J_i \| \mu^{(1)} \| j \Omega J(R)\rangle$

$$\cancel{\sigma_{KQ}(j', j, k) = \frac{1}{2\pi} c \sum \langle J_i \| \mu^{(1)} \| j \Omega J(R)\rangle \langle j' \Omega' J' | \mu}$$

$\sigma_{KQ}(j', j, k) =$ $\qquad \hat{\mu}^{*}_{(m)} = (-1)^m \mu_{-m}$ $\qquad \mu_1 = -\frac{\mu_x + i\mu_y}{\sqrt{2}}$

$\mu_{-1} = \frac{\mu_x - i\mu_y}{\sqrt{2}}$

$\langle J_i' M_i' |$

$$\langle j \Omega' J' M' | \hat{\mu}_{m_1} | J_i' M_i'\rangle = \langle J_i M_i | \hat{\mu}_{-m_1} | j \Omega' J' M'\rangle^{*} (-1)^{m_1}$$

$$= (-1)^{m_1} \begin{pmatrix} J_i' & 1 & J' \\ \bar{M}_i' & -m_1 & M' \end{pmatrix} \langle J_i' \| \mu^{(1)} \| j \Omega' J'\rangle^{*}$$

$$\langle 1 m_1\, 1 m_2 | k q\rangle = (-1)^q [k]^{1/2} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_2 & -q \end{pmatrix}$$

$$\cancel{(-1)^{k+q} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_2 & -q \end{pmatrix} = (-1)^{k} (-1)^{m_2} \begin{pmatrix} 1 & 1 & k \\ -m_1 & -m_2 & q \end{pmatrix}}$$

Ground state

$$|J_i M_i\rangle = \sum_{j_i \Omega_i} |j_i \Omega_i\rangle\, D^{J_i *}_{M_i, \Omega_i}(\alpha\beta 0)\, \frac{[J_i]^{1/2}}{\sqrt{4\pi}}\, \frac{\chi^{J_i}_{j_i \Omega_i}(R)}{R}$$

Partial wave

$$|j\Omega J M] = \frac{1}{R} \sum_{j''\Omega''} |j''\Omega'' J M\rangle\, f^{j\Omega J}_{j''\Omega''}(R)$$

BF basis

$$|j''\Omega'' J M\rangle = |j''\Omega''\rangle\, D^{(J)*}_{M, \Omega''}(\alpha\beta 0)\, \frac{[J]^{1/2}}{\sqrt{4\pi}}$$

BF dipole operator:

$$d^{SF}_{\mu} = \sum_t D^{(1)*}_{\mu t}\, d^{BF}_t$$

Integral

$$\iint D^{(J_i)}_{M_i \Omega_i}(\alpha,\beta,0)\, D^{(1)*}_{\mu t}(\alpha\beta 0)\, D^{(J)*}_{M\Omega''}(\alpha\beta 0)\, d\alpha\, d\cos\beta\, \frac{[J_i]^{1/2}[J]^{1/2}}{4\pi}$$

$$= [J_i]^{1/2}[J]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & \mu & M \end{pmatrix} \begin{pmatrix} J_i & 1 & J \\ \bar{\Omega}_i & t & \Omega'' \end{pmatrix}$$

$$\langle J_i M_i | d^{SF}_{\mu} | j\Omega J M\rangle =$$

$$\underbrace{\sum_{j_i\Omega_i}}_{\text{expansion initial state}}\ \sum_{t}\ \underbrace{\sum_{j''\Omega''}}_{\text{channel expansion}} \iint D^{(J_i)*}_{M_i\Omega_i}\, D^{(1)*}_{\mu t}\, D^{(J)}_{M\Omega''}\, d\alpha\, d\cos\beta\, \frac{[J_i]^{1/2}[J]^{1/2}}{4\pi}$$

$$\underbrace{\int \chi^{(J_i)*}_{j_i\Omega_i}(R)\, \langle j_i\Omega_i | d^{BF}_t | j''\Omega''\rangle\, f^{j\Omega J}_{j''\Omega''}(R)\, dR}$$

$$= \sum_{\Omega_i t \Omega''} [J_i]^{1/2}[J]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & \mu & M \end{pmatrix} \begin{pmatrix} J_i & 1 & J \\ \bar{\Omega}_i & t & \Omega'' \end{pmatrix} \sum_{j_i j''} M^{j\Omega J}_{J_i j_i \Omega_i; j''\Omega''}$$

$$= [J_i]^{1/2}[J]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & \mu & M \end{pmatrix} \underbrace{\sum_{\Omega_i t \Omega''} \begin{pmatrix} J_i & 1 & J \\ \bar{\Omega}_i & t & \Omega'' \end{pmatrix} \sum_{j_i j''} M^{j\Omega J}_{J_i j_i \Omega_i; j''\Omega''}}$$

$$= [J_i]^{1/2}[J]^{1/2} \begin{pmatrix} J_i & 1 & J \\ \bar{M}_i & \mu & M \end{pmatrix} \langle J_i \| d \| j\Omega J\rangle$$

$$\langle j\Omega J M | d^{SF}_{\mu} | J_i M_i\rangle = (-1)^{\mu} \langle J_i M_i | d_{-\mu} | j\Omega J M\rangle^{*} \qquad [d_{\mu}^{*} = (-1)^{\mu} d_{-\mu}]$$

$$\langle j\Omega J M | d_\mu | \hat{\rho}_{k_i q_i}(J_i, J_i') | d_{\mu'} | j'\Omega' J' M'\rangle$$

$$= \sum_{M_i M_i'} (-1)^{\mu} \langle j\Omega JM | d_\mu | J_i M_i\rangle \langle J_i' M_i' | d_{\mu'} | j'\Omega' J'M'\rangle [k_i]^{1/2} \begin{pmatrix} J_i & k_i & J_i' \\ \bar M_i & q_i & M_i'\end{pmatrix}$$

$$= \sum_{M_iM_i'} (-1)^\mu [J_i]^{1/2}[J_i']^{1/2}[J]^{1/2}[J']^{1/2}[k_i]^{1/2} \begin{pmatrix} J_i & 1 & J\\ \bar M_i & -\mu & M\end{pmatrix}\begin{pmatrix} J_i' & 1 & J' \\ \bar M_i' & \mu' & M'\end{pmatrix}\begin{pmatrix} J_i & k_i & J_i'\\ \bar M_i & q_i & M_i'\end{pmatrix}$$

$$\langle J_i \| d \| j\Omega J\rangle^* \langle J_i' \| d\| j'\Omega' J'\rangle$$

$$\sigma_{kq}(j,j',k) = \frac{4\pi^2\omega}{c}\sum_{\substack{kq \\ JJ'\ LM_L\ \Omega\Omega'}} E_{kq}\, C_{LM_L}(\hat k) \langle J_i\|d\|j\Omega J\rangle^*\langle J_i'\|d\|j'\Omega'J'\rangle$$

$$\left(\frac{-1}{2\pi}\right)\frac{1}{4\pi}\sum_{\substack{\mu\mu' q' MM' \\ m_j m_j'\ M_iM_i' \\ ll' \\ m_l m_l'}} (-1)^{m_l'} [K]^{1/2}[k]^{1/2}[k_i]^{1/2}[l]^{1/2}[l']^{1/2}[J_i]^{1/2}[J_i']^{1/2}[J]^{1/2}[J']^{1/2}$$

$$(-1)^\mu \begin{pmatrix} J_i & 1 & J\\ \bar M_i & -\mu & M\end{pmatrix}\begin{pmatrix}J_i' & 1 & J'\\ \bar M_i' & \mu' & M'\end{pmatrix}\begin{pmatrix}J_i & k_i & J_i'\\ \bar M_i & Q_i & M_i'\end{pmatrix}\langle 1\mu 1\mu' | kq'\rangle \begin{pmatrix} j & K & j'\\ \bar m_j & Q & m_j'\end{pmatrix}$$

$$\langle j m_j l m_l | JM\rangle\langle j' m_j' l' m_l' | J'M'\rangle\langle l m_l\, l' -m_l' | LM_L\rangle$$

$$U^{Jj}_{\Omega l}\, U^{J'j'}_{\Omega' l'} \langle l0\, l'0|L0\rangle \langle kq\, kq' | 00\rangle$$

$$\langle 1\mu 1\mu' | kq'\rangle = (-1)^{q'}[k]^{1/2}\begin{pmatrix}1&1&k\\ \mu&\mu'&-q'\end{pmatrix}$$

$$\langle kq\, kq' |00\rangle = \begin{pmatrix}k&k&0\\q&q'&0\end{pmatrix}$$

$$\langle l0\,l'0|L0\rangle = (-1)^{l-l'}[L]^{1/2}\begin{pmatrix} l&l'&L\\0&0&0\end{pmatrix}$$

$$\langle jm_j\, lm_l|JM\rangle = (-1)^{j-l+M}[J]^{1/2}\begin{pmatrix}j&l&J\\m_j&m_l&-M\end{pmatrix}$$

$$\langle lm_l\, l' -m_l' | LM_L\rangle = (-1)^{l-l'+M_L}[L]^{1/2}\begin{pmatrix} l&l'&L\\ m_l& -m_l' & -M_L\end{pmatrix}$$

$$U^{Jj}_{\Omega l} = \langle j\Omega\, l0|J\Omega\rangle\frac{[l]^{1/2}}{[J]^{1/2}} = (-1)^{j-l+\Omega}[l]^{1/2}\begin{pmatrix}j&l&J\\ \Omega&0&-\Omega\end{pmatrix}$$

$$\sigma_{kQ}(j'l) = \frac{4\pi^2\omega}{c} \sum_{\substack{kq \\ JJ'LM_L\Omega\Omega'}} E_{kq}\, C_{LM_L}(\hat{A}) \langle J_i \| d \| j\Omega J\rangle^* \langle J_i' \| d \| j'\Omega' J'\rangle$$

$$\frac{-1}{8\pi^2} \sum_{\substack{\mu\mu' q' MM' \\ m_j m_j' M_i M_i' \\ ll' m_l m_l'}} (-1)^{m_j' + J_i - M_i + J_i' - M_i' + J_i - M_i + q' + j - m_j + j - l + M + \mu + j' - l' + M' + l - l' + M_L + j - l + \Omega + j' - l' + \Omega' + l - l'}$$

$$[K]^{1/2}[k]^{\cancel{1/2}}[K_i]^{1/2}[l]^{\cancel{1/2}}[l']^{\cancel{1/2}}[J_i]^{1/2}[J_i']^{1/2}[J]^{\cancel{1/2}}[J']^{\cancel{1/2}}[L]$$

$$\cancel{[k]^{1/2}}\,\cancel{[J]^{1/2}}\,[J']^{1/2}\cancel{[L]^{1/2}}\,\cancel{[l]^{1/2}}\,\cancel{[l']^{1/2}}\,[L]^{1/2}$$

$$\begin{pmatrix} J_i & 1 & J \\ -M_i & -\mu & M \end{pmatrix}\begin{pmatrix} J_i' & 1 & J' \\ -M_i' & \mu' & M' \end{pmatrix}\begin{pmatrix} J_i & K_i & J_i' \\ -M_i & Q_i & M_i' \end{pmatrix}\begin{pmatrix} 1 & 1 & k \\ \mu & \mu' & -q' \end{pmatrix}\begin{pmatrix} j & K & j' \\ -m_j & Q & m_j' \end{pmatrix}$$

$$\begin{pmatrix} j & l & J \\ m_j & m_l & -M \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ m_j' & m_l' & -M' \end{pmatrix}\begin{pmatrix} l & l' & L \\ m_l & -m_l' & -M_L \end{pmatrix}\begin{pmatrix} j & l & J \\ \Omega & 0 & -\Omega \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ \Omega' & 0 & -\Omega' \end{pmatrix}$$

$$\begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} k & k & 0 \\ q & q' & 0 \end{pmatrix}$$

phase: $(-1)^{m_j' + \cancel{2J_i - 2M_i} + J_i' - M_i' + q' - j - m_j + M + 2j' + M' + M_L + \Omega + \Omega'}$

$= (-1)^{m_j' + J_i' - M_i' + q' + j + m_j + M + M' + M_L + 2j' + \Omega + \Omega' + \mu}$

factor: $[K]^{1/2}[k][K_i]^{1/2}[l][l'][J_i]^{1/2}[J_i']^{1/2}[J][J'][L]$

phase: $(-1)^{\boxed{\mu + J_i' - M_i' + q'}} + \boxed{M + M' \quad (-1)^{m_j' + j + m_j + 2j' + \Omega + \Omega' + M_L}}$

$m_l = M - m_j$

(-1)

$M + m_j = 2J - M + m_j = 2J - m_l$

M'

$M_L + m_j' = m_l \quad \times$

$M + M_L + m_j + m_j' = 2J$

$(-1)^{2J + M' + j + \Omega + \underbrace{2j' + \Omega'}_{-\Omega'}} = (-1)^{\boxed{2J + M' + j + \Omega - \Omega'}}$

van p6 (11)

$$\sigma_{K\alpha}(JJ';\lambda) = -\frac{\omega}{2c} \sum_{\substack{\lambda_2 L M_2 \\ JJ'\Omega\Omega'}} E_{\lambda_2}\, \varepsilon_{LM_2}(\hat{\lambda}) \langle J_i \| d \| J \Omega J \rangle^{*} \langle J_i' \| d \| J'\Omega' J' \rangle (-1)^{\Omega-\Omega'}$$

$$\sum_{\substack{q'MM' \\ m_j m_j' m_2 m_2'}} \sum_{\ell\ell'} [K]^{1/2} [K_i]^{1/2} [J_i]^{1/2} [J_i']^{1/2} [\lambda][\ell][\ell'][J][J'][L] \times B$$

$$B = (-1)^{\cancel{\dots}\, 2J + M' + j} \begin{pmatrix} j & \ell & J \\ \Omega & 0 & -\Omega \end{pmatrix} \begin{pmatrix} j' & \ell' & J' \\ \Omega' & 0 & -\Omega' \end{pmatrix} \begin{pmatrix} \ell & \ell' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \lambda & k & 0 \\ q & q' & 0 \end{pmatrix}$$

$$\begin{pmatrix} j & K & j' \\ -m_j & Q & m_j' \end{pmatrix} \begin{pmatrix} j & \ell & J \\ m_j & m_\ell & -M \end{pmatrix} \begin{pmatrix} j' & \ell' & J' \\ m_j' & m_\ell' & -M' \end{pmatrix} \begin{pmatrix} \ell & \ell' & L \\ m_\ell & -m_\ell' & -M_2 \end{pmatrix} \times A$$

$$A = \sum_{\mu\mu' M_i M_i'} (-1)^{\mu + J_i' - M_i' + q'} \begin{pmatrix} J_i & 1 & J \\ -M_i & -\mu & M \end{pmatrix} \begin{pmatrix} J_i' & 1 & J' \\ -M_i' & \mu' & M' \end{pmatrix} \begin{pmatrix} J_i & K_i & J_i' \\ -M_i & Q_i & M_i' \end{pmatrix} \begin{pmatrix} 1 & 1 & k \\ \mu & \mu' & -q' \end{pmatrix}$$

$M_i \to -M_i$ ~~...~~ $M_i' \to -M_i'$
$\mu \to -\mu$
$\mu' \to -\mu'$

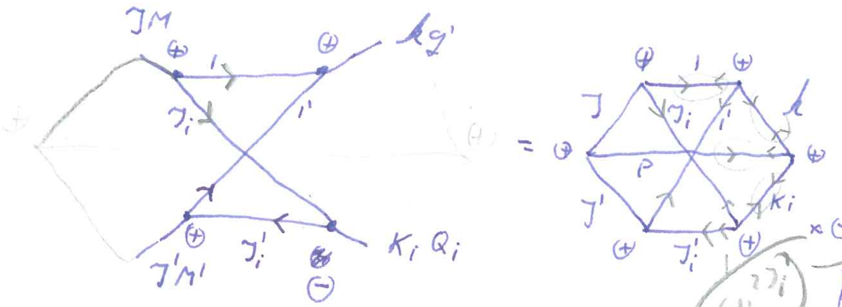
$$\sum_{\mu\mu' M_i M_i'} (-1)^{J_i' + M_i' + \mu + q'} \begin{pmatrix} J_i & 1 & J \\ +M_i & \mu & M \end{pmatrix} \begin{pmatrix} J_i' & 1 & J' \\ +M_i' & -\mu' & M' \end{pmatrix} \begin{pmatrix} J_i & K_i & J_i' \\ M_i & Q_i & -M_i' \end{pmatrix} \begin{pmatrix} 1 & 1 & k \\ +\mu & +\mu' & +q' \end{pmatrix} (-1)^{k}$$

$\mu + q' = -\mu' \quad\Rightarrow\quad$ ~~...~~ $-\mu' = 2 + \mu' = 1 + 1 + \mu'$

$$= \cancel{(-1)^{1+k} \sum_{\mu\mu' M_i M_i'} \begin{pmatrix} J_i & 1 & J \\ M_i & \mu & M \end{pmatrix} \dots \begin{pmatrix} J_i' & 1 & J' \\ M_i & \mu & M \end{pmatrix} \begin{pmatrix} J_i & 1 & J' \\ M_i' & -\mu' & M' \end{pmatrix}}$$

$$= (-1)^{1+k} \sum_{\substack{\mu\mu' \\ M_i M_i'}} \begin{pmatrix} J_i & 1 & J \\ M_i & \mu & M \end{pmatrix} (-1)^{1+\mu'} \begin{pmatrix} J_i' & 1 & J' \\ M_i' & -\mu' & M' \end{pmatrix} (-1)^{J_i' + M_i'} \begin{pmatrix} J_i & K_i & J_i' \\ M_i & Q_i & -M_i' \end{pmatrix} \begin{pmatrix} 1 & 1 & k \\ \mu & \mu' & q' \end{pmatrix}$$

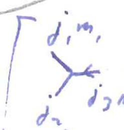
$= (-1)^{1+k}$



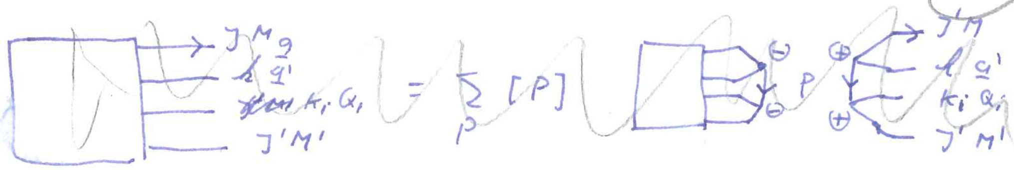
$$\begin{Bmatrix} 1 & 1' & k \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix}$$

$\times (-1)^{J_i + J_i' + K_i}$ to last eq

$(-1)^{2J_i}$



$$= \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix} (-1)^{j_3 - m_3}$$



$(-1)^{2J'} \begin{pmatrix} J' & P & J \\ -M' & p & M \end{pmatrix} (-1)^{J'-M'}$

$$\begin{pmatrix} P & K_i & k \\ p & Q_i & q' \end{pmatrix}$$

$$A = (-1)^{1+k+J_i+J_i'+K_i+J'-M'} \begin{Bmatrix} 1 & 1' & k \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix} \begin{pmatrix} J' & P & J \\ -M' & p & M \end{pmatrix} \begin{pmatrix} k & P & K_i \\ q' & p & Q_i \end{pmatrix} [P]$$

$$B = \sum_{q'} (-1)^{2J+j+1+k+J_i+J_i'+K_i+J'} [P]$$

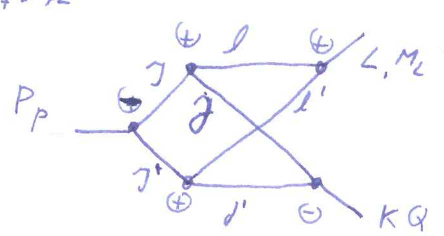
$$\begin{pmatrix} j & l & J \\ \Omega & 0 & -\Omega \end{pmatrix} \begin{pmatrix} j' & l' & J' \\ \Omega' & 0 & -\Omega' \end{pmatrix} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} k & k & 0 \\ q & q' & 0 \end{pmatrix} \begin{Bmatrix} 1 & 1' & k \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix} \begin{pmatrix} k & P & K_i \\ q' & p & Q \end{pmatrix} \times C$$

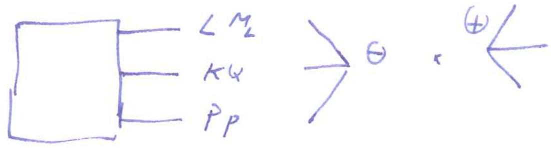
$$C = \sum_{\substack{M M' \\ m_j m_j' \\ m_l m_l'}} \begin{pmatrix} j & K & j' \\ -m_j & Q & m_j' \end{pmatrix} \begin{pmatrix} j & l & J \\ m_j & m_l & -M \end{pmatrix} \begin{pmatrix} j' & l' & J' \\ m_j' & m_l' & -M' \end{pmatrix} \begin{pmatrix} l & l' & L \\ m_l & -m_l' & -M_L \end{pmatrix} \begin{pmatrix} J' & P & J \\ -M' & p & M \end{pmatrix}$$

$$m_j \quad Q \quad m_j' \qquad \underbrace{-m_j \ -m_l \ -M}_{(-1)^{j+l+J}} \qquad m_j' \ m_l' \ +M' \qquad \underbrace{-m_l \ -m_l' \ -M_L}_{(-1)^{l+l'+L}} \qquad +M' \ p \ M$$

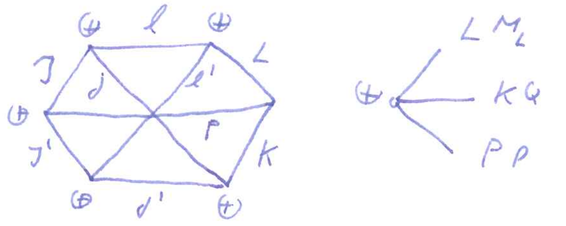
$$= (-1)^{j+l+J+l+l'+L} \sum_{\substack{M M' \\ m_j m_j' \\ m_l m_l'}} \begin{pmatrix} j & K & j' \\ m_j & Q & m_j' \end{pmatrix} \begin{pmatrix} j & l & J \\ m_j & m_l & M \end{pmatrix} \begin{pmatrix} j' & l' & J' \\ m_j' & m_l' & M' \end{pmatrix} \begin{pmatrix} l & l' & L \\ m_l & m_l' & M_L \end{pmatrix} \begin{pmatrix} J' & P & J \\ M' & p & M \end{pmatrix}$$

$= (-1)^{j+l+J+l+l'+L}$





$= (-1)^{?\, j+2l+l'+J+L+j+j'+K}$



$$\begin{Bmatrix} l & l' & L \\ J & J' & P \\ j & j' & K \end{Bmatrix} \qquad \begin{pmatrix} K & L & P \\ Q & M_L & p \end{pmatrix}$$

---

total phase B:

$$(-1)^{\underbrace{3J+3j}+1+k+J_i+J_i'+K_i+J'+\underline{2}l+l'+L+j'+K}$$

$$= (-1)^{\underline{J}+\underline{j}+1+k+J_i+J_i'+K_i+\underline{J'}+\underline{l'}+L+\underline{j'}+K}$$

$$= (-1)^{j+j'+l'+1+k+J+J'+J_i+J_i'+K_i+K+L}$$

$$B = (-1)^{j+j'+l'+1+h+J+J'+J_i+J_i'+K_i+K+L}\,[P]$$

$$\begin{pmatrix} j & l & J \\ \Omega & 0 & -\Omega \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ \Omega' & 0 & -\Omega' \end{pmatrix}\begin{pmatrix} h & h & 0 \\ q & -q & 0 \end{pmatrix}\begin{Bmatrix} 1 & 1 & h \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix}\begin{pmatrix} h & P & K_i \\ -q & p & Q_i \end{pmatrix}\begin{pmatrix} K & L & P \\ Q & M_L & p \end{pmatrix} * D$$

$$D = \begin{Bmatrix} l & l' & L \\ J & J' & P \\ j & j' & K \end{Bmatrix}\begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} = \begin{pmatrix} j & l & J \\ m_j & \underset{0}{0} & M_J \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ m_j' & \underset{0}{0} & M_J' \end{pmatrix}\begin{pmatrix} K & L & P \\ M_K & \underset{0}{0} & M_P \end{pmatrix}\begin{pmatrix} j & j' & K \\ m_j & m_j' & M_K \end{pmatrix}\begin{pmatrix} J & J' & P \\ M_J & M_J' & M_P \end{pmatrix}$$

$$\sum_l [l]\begin{pmatrix} j & l & J \\ \Omega & 0 & -\Omega \end{pmatrix}\begin{pmatrix} j & l & J \\ m_j & \underset{0}{0} & M_J \end{pmatrix} = \delta_{m_j,\Omega}\,\delta_{M_J,-\Omega}$$

$$\sum_{l'} (-1)^{l'}[l']\begin{pmatrix} j' & l' & J' \\ \Omega' & 0 & -\Omega' \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ m_j' & m_j' & M_J' \end{pmatrix} = (-1)^{j'+J'}\sum_{l'}[l']\begin{pmatrix} j' & l' & J' \\ -\Omega' & 0 & \Omega' \end{pmatrix}\begin{pmatrix} j' & l' & J' \\ m_j' & m_j' & M_J' \end{pmatrix}$$

$$= (-1)^{j'+J'}\,\delta_{m_j',-\Omega'}\,\delta_{M_J',\Omega'}$$

$$\left.\begin{matrix} m_j = \Omega & m_j' = -\Omega' \\ M_J = -\Omega & M_J' = \Omega' \end{matrix}\right\}\quad \begin{aligned} M_P &= -M_J - M_J' = \Omega - \Omega' \equiv w \\ M_K &= -m_j - m_j' = -\Omega + \Omega' = -w \\ M_L'' &= -M_K - M_P = 0 \end{aligned}$$

$\boxed{21/6/2002}$

$$\sigma_{KQ}(jj'h) = \frac{w}{2c}\sum_{\substack{hqLM_L \\ JJ'\Omega\Omega'}} E_{hq}\, C_{LM_L}(h)\,\langle J_i \| d \| j\Omega J_i\rangle^* \langle J_i' \| d \| j'\Omega' J_i'\rangle (-1)^{\Omega-\Omega'}$$

$$\boxed{\begin{aligned} w &= \Omega - \Omega' \\ p &= q - Q_i \end{aligned}}$$

$$\sum_P [K]^{1/2}[K_i]^{1/2}[J_i]^{1/2}[J_i']^{1/2}[h][J][J'][L][P]$$

$$\begin{Bmatrix} 1 & 1 & h \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix}\begin{pmatrix} h & K_i & P \\ -q & Q_i & p \end{pmatrix}\begin{pmatrix} K & L & P \\ Q & M_L & p \end{pmatrix}\begin{pmatrix} K & L & P \\ -w & 0 & w \end{pmatrix}\begin{pmatrix} j & j' & K \\ \Omega & -\Omega' & -w \end{pmatrix}\begin{pmatrix} J & J' & P \\ -\Omega & \Omega' & w \end{pmatrix}$$

$$\begin{pmatrix} h & h & 0 \\ q & -q & 0 \end{pmatrix} \times (-1)^{J_i - J_i' + j + J + L + K + P}$$

$$(-1)^{j+j'+h+J+J'+J_i+J_i'+K_i+K+L+h+K_i+P+j'+J'}$$

$$= (-1)^{j+j'+2h+J+J'+J_i+J_i'+2K_i+K+L+P+j'+J'}$$

$$= (-1)^{\cancel{2h}+2K_i+J_i+J_i'+j+j'+J+J'+L+K+P+j'+J'}$$

$$\cancel{2h} \to 0 \qquad \underbrace{\qquad\qquad}_{\cancel{l+l'}}$$

$\times(-1)^{2J_i'}$

from page 11

$$(-1)^{J_i - J_i' + j + 2j' + J + 2J' + L + K + P}$$

$$\boxed{= (-1)^{J_i - J_i' + j + J + L + K + P}}$$

$$\rho^*_{KQ}(j_1 j_2) = \rho_{K,-Q}(j_2 j_1)\,(-1)^{j_1 - j_2 + Q}$$

$2j - j'$

$2j + 2j' + j'$

$$\sigma^*_{KQ}(jj') = (-1)^{j - j' + Q}\,\sigma_{K,-Q}$$

$$= \frac{\omega}{2c} \sum_{\substack{\lambda q L M_L \\ J J' \Omega \Omega'}} E_{\lambda q}\, C_{LM_L}(\hat{\lambda})\, \langle J_i \| d \| j \Omega J\rangle^* \langle J_i' \| d \| j' \Omega' J'\rangle$$

$W = \Omega - \Omega'$

$p = q - Q_i$

$$(-1)^{\Omega - \Omega'} \sum_P [K]^{1/2} [K_i]^{1/2} [J_i]^{1/2} [J_i']^{1/2} [\lambda][J][J'][L][P]$$

$$\begin{Bmatrix} 1 & 1 & \lambda \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix} \begin{pmatrix} \lambda & K_i & P \\ -q & +Q_i & p \end{pmatrix} \begin{pmatrix} K & L & P \\ Q & -M_L & -p \end{pmatrix} \begin{pmatrix} K & L & P \\ -W & 0 & W \end{pmatrix} \begin{pmatrix} j & j' & K \\ \Omega & -\Omega' & -W \end{pmatrix} \begin{pmatrix} J & J' & P \\ -\Omega & \Omega' & W \end{pmatrix}$$

$$\begin{pmatrix} \lambda & \lambda_0 \\ q & -q_0 \end{pmatrix} \quad \boxed{(-1)^{J_i - J_i' + \cancel{} j' + J + Q}}$$

$$\langle J_i \| d \| j \Omega J\rangle = (-1)^{2J_i + 1 + J} \langle J_i \| d \| j \Omega J\rangle^{\text{roncero}}$$

roncero phase:

$2J + J' = -J'$

$$(-1)^{J_i - J_i' + j' + J + Q + 2J_i + 2J_i' + J + J'}$$

$$= (-1)^{J_i' - J_i + 2J + J' + j' + Q}$$

$$\boxed{= (-1)^{J_i' - J_i - J' + j' + Q}}$$

$\sigma^*_{KQ}(jj') =$

Gerrit: $\langle J_i \| d \| j \Omega J\rangle = \sum \begin{pmatrix} J_i & 1 & J \\ -\Omega_i & + & \Omega' \end{pmatrix} M_{\Omega_i \Omega'}$

$$= (-1)^{J_i - \Omega_i} \begin{pmatrix} J_i & 1 & J \\ -\Omega_i & + & \Omega' \end{pmatrix} M_{\Omega_i \Omega'}$$

$$= (-1)^{2J_i + 1 + J} \underline{(-1)^{-\Omega_i} \begin{pmatrix} J & 1 & J_i \\ \Omega' & + & -\Omega_i \end{pmatrix} M_{\Omega_i \Omega'}}$$

$$= (-1)^{2J_i + 1 + J} \langle J_i \| d \| j \Omega J\rangle_{\text{roncero}}$$

Unpolarized initial state $k_i = Q_i = 0$

single product $d = d'$

$$\Downarrow \quad J_i = J_i' \quad P = h \quad p = q$$

$$\begin{pmatrix} h & k_i & P \\ -q & Q_i & p \end{pmatrix}\begin{pmatrix} h & h & 0 \\ q & -q & 0 \end{pmatrix} = \begin{pmatrix} h & h & 0 \\ q & -q & 0 \end{pmatrix}^2 = \underline{[h]^{-1}}$$

$$\begin{Bmatrix} 1 & 1 & h \\ J & J' & h \\ J_i & J_i & 0 \end{Bmatrix} = (-1)^{1+h+J+J_i} \underline{[h]^{-1/2}} \underline{[J_i]^{-1/2}} \begin{Bmatrix} 1 & 1 & h \\ J' & J & J_i \end{Bmatrix}$$

phase (Roncero convention):

$$(-1)^{\underline{J_i' - J_i} \; \underline{-J'} + d' + \underline{Q} \; + \underline{1} + \underline{h} + \underline{J} + J_i} \times (-1)^{\Omega - \Omega'}$$

$$= (-1)^{1 + Q + h + J - J' + J_i + d'}$$

$$= (-1)^{1 - Q - h - J + J' - J_i \underbrace{-d'}} \underbrace{\times (-1)^{\Omega - \Omega'}}_{(-1)^{d' - \Omega - \Omega'}}$$

$(-1)^{-d'}$

unpolarized initial state

$$\boxed{\begin{aligned} \sigma^*_{KQ}(d\,d') &= \frac{\omega}{2\varepsilon} \sum_{\substack{h q L M_L \\ J J' \Omega \Omega'}} E_{hq} \; C_{LM_L}(\hat{k}) \langle J_i \| d \| d\Omega J \rangle^*_{\text{Roncero}} \langle J_i' \| d \| d\Omega' J' \rangle_{\text{Roncero}} \\ &\quad \times (-1)^{d' - \Omega - \Omega'} [J]^{1/2} [J']^{1/2} \\ &\quad [h]\cdot[P]\cdot[h]^{-1}\cdot[J_i]^{-1/2} \\ &\quad [K]^{1/2} [J_i]^{1/2} [h]^{+1/2} [J]^{1/2} [J']^{1/2} [L] \; (-1)^{1 - Q - h - J + J' - J_i} \\ &\quad \begin{Bmatrix} 1 & 1 & h \\ J & J' & J_i \end{Bmatrix} \begin{pmatrix} K & L & h \\ Q & -M_L & -q \end{pmatrix} \begin{pmatrix} K & L & h \\ -w & 0 & w \end{pmatrix} \begin{pmatrix} d & d' & k \\ \Omega & -\Omega' & -w \end{pmatrix} \begin{pmatrix} J & J' & h \\ -\Omega & \Omega' & w \end{pmatrix} \\ &\quad \hookrightarrow M_L = Q - q \qquad w = \Omega - \Omega' \end{aligned}}$$

$$\rho_{0,0} = \sum_{M_i} |J_i M_i\rangle\langle J_i M_i| \begin{pmatrix} J_i & 0 & J_i \\ -M_i & 0 & M_i \end{pmatrix} (-1)^{J_i - M_i}$$

$$\hookrightarrow \frac{(-1)^{J_i - M_i}}{[J_i]^{1/2}} \cdot (-1)^{J_i - M_i} = [J_i]^{-1/2}$$

$$= [J_i]^{-1/2} \sum_i |J_i M_i\rangle\langle J_i M_i|$$

multiply σ^*_{KQ} with: to get Eq (14) JPC 98 3307 (1994)

~~Roncero~~

Roncero: initial $[J_i]^{-1} \sum_M |J_i M_i\rangle\langle J_i M_i|$ → extra $[J]_i^{-1/2}$

~~K_{LM} $2\pi^{1/2}$~~ $C_{LM} = \underline{2\pi^{1/2}} [L]^{-1/2} Y_{LM}$ → extra $2\pi^{1/2}[L]^{-1/2}$

Energy norm → → extra 2π

Direct dissociation with polarized initial state

~~Notation~~

$$\langle J_i \| \underline{d} \| j\Omega J \rangle = \sum_{\Omega_i t \Omega''} (-1) \begin{pmatrix} J_i & 1 & J \\ \Omega_i & t & \Omega'' \end{pmatrix} \sum_{j_i j''} M^{(j\Omega J)}_{J_i j_i \Omega_i; j''\Omega''}$$

Assume Ω is good quantum number; M is J independent:

$$M^{(j\Omega J)}_{j_i\Omega_i; j''\Omega''} \approx M^{j\Omega J_i}_{J_i j_i \Omega_i; j''\Omega} \, \delta_{\Omega'',\Omega}$$

$$\langle J_i \| \underline{d} \| j\Omega J \rangle = \sum_{\Omega_i t} \begin{pmatrix} J_i & 1 & J \\ \Omega_i & t & \Omega \end{pmatrix} \underbrace{\sum_{j_i j''} M^{(j\Omega J_i)}_{J_i j_i \Omega_i; j''\Omega}}_{\tilde{M}_{J_i\Omega_i j\Omega}}$$

from p 13

$$\sigma_{KQ}(jj'\lambda) = \frac{\omega}{2c} \sum_{\substack{\lambda q L M_L \\ \Omega\Omega' \\ \Omega_i t \Omega_i' t'}} E_{\lambda q} \, \epsilon M^{(\lambda)} \sum_{jj''} \tilde{M}^*_{J_i\Omega_i j\Omega} \tilde{M}_{J_i'\Omega_i' j\Omega'} (-1)^{\Omega-\Omega'} \begin{pmatrix} j & j' & K \\ \Omega & \Omega' & -w \end{pmatrix}$$

$$\sum_P [K]^{1/2} [K_i]^{1/2} [J_i]^{1/2} [J_i']^{1/2} [\lambda] [\cancel{JJ'}] [L][P]$$

$$\begin{pmatrix} \lambda & K_i & P \\ -q & Q_i & p \end{pmatrix} \begin{pmatrix} K & L & P \\ Q & M_L & p \end{pmatrix} \begin{pmatrix} K & L & P \\ -w & 0 & w \end{pmatrix} \begin{pmatrix} \lambda & \lambda & 0 \\ q & -q & 0 \end{pmatrix} (-1)^{J_i - J_i' + \cancel{t} + L + K + P} \times A$$

$$A = \sum_{J_0} \begin{Bmatrix} 1 & 1 & \lambda \\ J & J' & P \\ J_i & J_i' & K_i \end{Bmatrix} \begin{pmatrix} J & J' & P \\ -\Omega & \Omega' & w \end{pmatrix} \begin{pmatrix} J_i & 1 & J \\ \Omega_i & t & \Omega \end{pmatrix} \begin{pmatrix} J_i' & 1 & J' \\ \Omega_i' & t' & \Omega' \end{pmatrix} [J][J'] (-1)^{j+J}$$

$$(-1)^{J_i-\Omega_i} \begin{pmatrix} J_i & 1 & J \\ -\Omega_i & t & \Omega \end{pmatrix}$$

$$= (-1)^{J_i - \Omega_i - J_i - 1 - J} \begin{pmatrix} J_i & 1 & J \\ \Omega_i & -t & -\Omega \end{pmatrix}$$

$$= (-1)^{-\Omega_i + 1 - J} \begin{pmatrix} J_i & 1 & J \\ \Omega_i & -t & -\Omega \end{pmatrix}$$

$$= (-1)^{j - J_i - 1}$$

$$= (-1)^{j-\Omega_i+1} \sum_{JJ'} \begin{Bmatrix} P & J & J' \\ K_i & J_i & J_i' \\ \lambda & 1 & 1 \end{Bmatrix} \begin{pmatrix} P & J & J' \\ w & -\Omega & \Omega' \end{pmatrix} \begin{pmatrix} J & J_i & 1 \\ -\Omega & \Omega_i & -t \end{pmatrix} \begin{pmatrix} J' & J_i' & 1 \\ \Omega' & -\Omega_i' & t' \end{pmatrix} (-1)^{J_i' - \Omega_i'}$$

$$= \begin{pmatrix} P & K_i & \lambda \\ w & \delta & t - t' \end{pmatrix} \begin{pmatrix} K_i & J_i & J_i' \\ \Omega_i' - \Omega_i & \Omega_i & -\Omega_i' \end{pmatrix} \begin{pmatrix} \lambda & 1 & 1 \\ t - t' & -t & t' \end{pmatrix}$$

$\Omega - \Omega'$, $\Omega_i' - \Omega_i$

$$\delta = t' - t - w \qquad t = \Omega_i - \Omega$$
$$= \Omega_i' - \Omega_i \qquad t' = \Omega_i' - \Omega'$$
$$t - t' = \Omega_i - \Omega_i' - (\Omega - \Omega')$$

$$A = (-1)^{j-\Omega_i+1+J_i'-\Omega_i'} \begin{pmatrix} p & k_i & k \\ w & \Omega_i'-\Omega_i & \overset{t-t'}{\cancel{x}} \end{pmatrix} \begin{pmatrix} k_i & J_i & J_i' \\ \Omega_i'-\Omega_i & \Omega_i & -\Omega_i' \end{pmatrix} \begin{pmatrix} d & 1 & 1 \\ t-t' & -t & t' \end{pmatrix}$$

$$t = \Omega_i - \Omega$$

Pinch dissociation with polarized initial state

$$\sigma_{kQ}(jj'h) = \frac{v}{2c} \sum_{\substack{lg\,LM_L \\ \Omega\Omega'}} E_{lg}\, C_{LM_L}(\hat{L})\, \tilde{M}^*_{J_i\Omega_i;j\Omega}\, \tilde{M}_{J_i'\Omega_i';j'\Omega'}$$

$$(-1)^{\Omega-\Omega'} \begin{pmatrix} j & j' & k \\ \Omega & -\Omega' & -w \end{pmatrix}$$

$$\sum_P [k]^{1/2}[k_i]^{1/2}[J_i]^{1/2}[J_i']^{1/2}[l][LkP]$$

$$\begin{pmatrix} l & k_i & P \\ -g & Q_i & P \end{pmatrix} \begin{pmatrix} k & L & P \\ Q & M_L & p \end{pmatrix} \begin{pmatrix} k & L & P \\ -w & 0 & w \end{pmatrix} \begin{pmatrix} l & l & 0 \\ g & -g & 0 \end{pmatrix} (-1)^{J_i - \cancel{J_i'} + L + k + P}$$

$= P$

$$(-1)^{j - \Omega_i + 1 + \cancel{J_i'} - \Omega_i'} \begin{pmatrix} P & k_i & l \\ w & \Omega_i' - \Omega_i & t - t' \end{pmatrix} \begin{pmatrix} k_i & J_i & J_i' \\ \Omega_i' - \Omega_i & \Omega_i & -\Omega_i' \end{pmatrix} \begin{pmatrix} l & 1 & 1 \\ t - t' & -t & t' \end{pmatrix}$$

$\times (-1)^{2j'}$ ↳ underwood

$w = \Omega - \Omega'$

$t = \Omega_i - \Omega$

$t' = \Omega_i' - \Omega'$

$(-1)^{\Omega - \Omega' + \Omega_i - \Omega_i'}$

$\Omega = \Omega_i - t$

$\Omega' = \Omega_i' - t'$

$\Omega - \Omega_i = -t$

Ω_i

$-\Omega' - \Omega_i'$

Underwood:

$$M^{\Omega\Omega_0}_{jj_a j_b}(g) = \int \chi^{J*}_{J}(R) \langle \Omega j j_a j_b | d_g | \Omega_0 \rangle \chi^{J_0}_{J_0}(r)\, dr$$

phase underwood: $(-1)^{-J_0 + g + L + 2j_a - j_a' + j_b + j + j'}$

$= (-1)^{-J_i + Q_i + L + 2j_a' - j_a + j_b^* + j' + j}$

Gernit $(-1)^{\Omega - \Omega' - \Omega_i - \Omega_i' + J_i + L + k + P + j + 1 - j_a - 2j_a' + j_b + j' + k}$

extra $(-1)^{t + t'} = \Omega_i + \Omega_i' - \Omega - \Omega'$, $(-1)^{2\Omega'} = (-1)^{2j'}$ Underwood was right 5 apr 06

extra ~~$\sigma_{k_i Q}$ $j_a' + Q$~~

$\sigma^+_{kQ}(j_a j_a') = \cancel{(-1)^{j_a + j_a' + Q}}\, \cancel{\sigma_{k Q}}\, (-1)^{j - j' + Q}$

$q = 0 \Rightarrow p = -Q_i = -Q - M_L$

uncoupled fragments

$$\sigma_{j_a m_a j_b m_b; j_a' m_a' j_b' m_b'} = \frac{4\pi^2\omega}{c} \langle \hat{E}^- j_a m_a j_b m_b | \underline{e}\cdot\underline{d} | \rho_{K_i G_i}(J_i J_i') | \underline{e}^*\cdot\underline{d} | \hat{E}^- j_a' m_a' j_b' m_b' \rangle$$

coupled fragments

$$\sigma_{(j_a j_b) j m; (j_a' j_b') j' m'} = \sum_{\substack{m_a m_b \\ m_a' m_b'}} \sigma_{j_a m_a j_b m_b; j_a' m_a' j_b' m_b'} \langle j_a m_a j_b m_b | j m \rangle \langle j_a' m_a' j_b' m_b' | j' m' \rangle$$

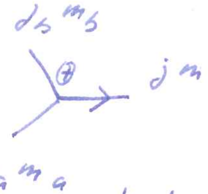
Irreducible components

$$\sigma_{KG}[(j_a j_b) j; (j_a' j_b') j'] = \sigma_{(j_a j_b) j m; (j_a' j_b') j' m'} [K]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m} & G & m' \end{pmatrix}$$

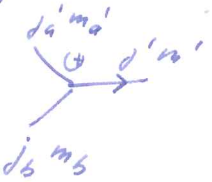
Irreducible component fragment (a); polarization of fragment (b) not detected

$$\sigma_{KG}(j_a j_a'; j_b) = \sum_{\substack{m_a m_a' \\ m_b}} \sigma_{j_a m_a j_b m_b; j_a' m_a' j_b m_b} [K]^{1/2} \begin{pmatrix} j_a & K & j_a' \\ \bar{m}_a & G & m_a' \end{pmatrix}$$

$$= \sum_{m_a m_a' m_b} \sum_{\substack{jm \\ j'm'}} \sigma_{(j_a j_b) j m; (j_a' j_b) j' m'} \langle j_a m_a j_b m_b | j m \rangle \langle j_a' m_a' j_b m_b | j' m' \rangle [K]^{1/2} \begin{pmatrix} j_a & K & j_a' \\ \bar{m}_a & G & m_a' \end{pmatrix}$$

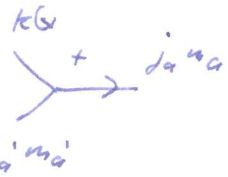
$\langle j_a m_a j_b m_b | j m \rangle =$ 

$\boxed{(-1)^{2j_a} [j]^{1/2}}$

$\langle j_a' m_a' j_b m_b | j' m' \rangle =$ 

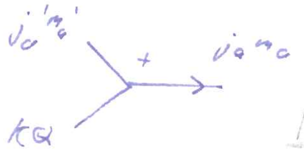
$\boxed{(-1)^{j_a' - j_b + j'} [j']^{1/2}}$

$$[K]^{1/2} \begin{pmatrix} j_a & K & j_a' \\ \bar{m}_a & G & m_a' \end{pmatrix} = (-1)^{2j_a} (-1)^{j_a + m_a} \begin{pmatrix} j_a & K & j_a' \\ -m_a & G & m_a' \end{pmatrix} [K]^{1/2}$$

$= \cancel{\text{cel}}$ 

$(-1)^{2j_a} [K]^{1/2}$

$= \cancel{(-1)^{2j_a + j_a' - j_b + j' + 2j_a} [j]^{1/2} [j']^{1/2} [K]^{1/2}}$

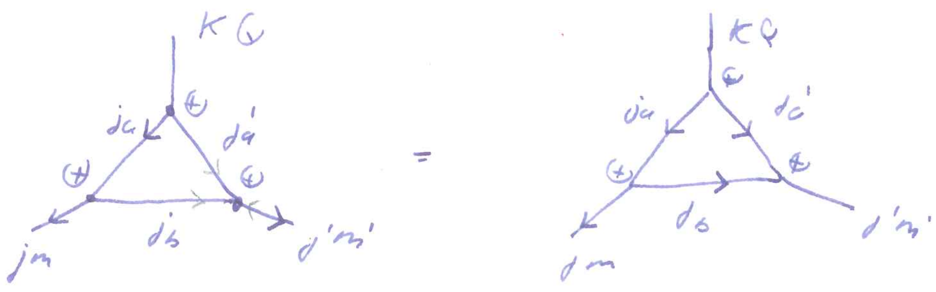
$=$ 

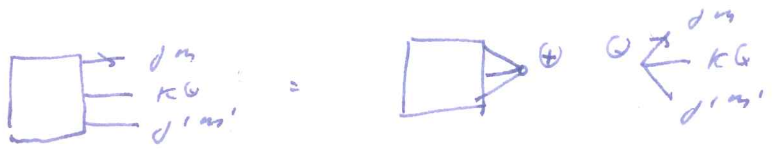
$\boxed{(-1)^{K + j_a' - j_a} [K]^{1/2}}$

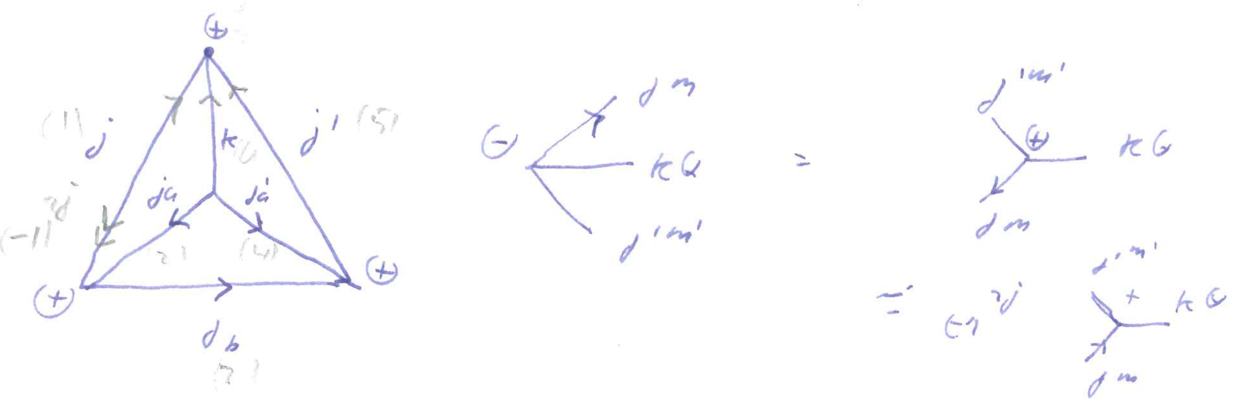
(24/6/2007) 20 2c

$$= (-1)^{2j_a + \underbrace{j_a' - j_b + j'}_{} + \underbrace{k + j_a'}_{} - j_a} [j]^{1/2} [j']^{1/2} [K]^{1/2}$$

$$= (-1)^{+j_a + 2j_a' - j_b + j' + k} [j]^{1/2} [j']^{1/2} [K]^{1/2}$$







$$\cancel{(-1)^{2j}} \qquad \cancel{(-1)^{j}} \begin{pmatrix} j & K & j' \\ \bar{m} & Q & m' \end{pmatrix}$$

$$= (-1)^{-j_a - 2j_a' + j_b + j' + k} [j]^{1/2} [j']^{1/2} \begin{Bmatrix} j & j_a & j_b \\ j_a' & j' & k \end{Bmatrix} [K]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m} & Q & m' \end{pmatrix}$$

$$\boxed{\sigma_{KQ}(j_a j_a'; j_b) = \sum_{jj'} \sum_{mm'} \sigma_{(j_a j_b) jm; (j_a' j_b) j'm'} [K]^{1/2} \begin{pmatrix} j & K & j' \\ \bar{m} & Q & m' \end{pmatrix} (-1)^{-j_a - 2j_a' + j_b + j' + K} [j]^{1/2} [j']^{1/2} \begin{Bmatrix} j & j_a & j_b \\ j_a' & j' & K \end{Bmatrix}}$$

$$\sigma_{KQ}(j_a, j_a'; j_s) = \sum_{j j'} \frac{\omega}{2c} \sum_{\substack{\ell_2 \\ L M_L \\ \Omega\Omega'}} E_{\ell q}\, C_{L M_L}\, \tilde{M}^*_{J_i \Omega_i; j\Omega}\, \tilde{M}_{J_i' \Omega_i'; j'\Omega'}$$

$$\sum_P [K]^{1/2} [K_i]^{1/2} [J_i]^{1/2} [J_i']^{1/2} [\ell] [L] [P] [j]^{1/2} [j']^{1/2}$$

$$\begin{pmatrix} \ell & K_i & P \\ -q & Q_i & p \end{pmatrix} \begin{pmatrix} k & L & P \\ Q & M_L & p \end{pmatrix} \begin{pmatrix} K & L & P \\ -w & 0 & w \end{pmatrix} \begin{pmatrix} P & K_i & \ell \\ w & \Omega_i'-\Omega_i & t-t' \end{pmatrix} \begin{pmatrix} K_i & J_i & J_i' \\ \Omega_i'-\Omega_i & \Omega_i & -\Omega_i' \end{pmatrix}$$

$$\begin{pmatrix} \ell & 1 & 1 \\ t-t' & -t & t' \end{pmatrix} \begin{pmatrix} j & j' & K \\ \Omega & -\Omega' & -w \end{pmatrix} \begin{pmatrix} \ell & \ell & 0 \\ q & -q & 0 \end{pmatrix} \begin{Bmatrix} j & j_a & j_s \\ j_a' & j' & K \end{Bmatrix}$$

$$(-1)^{J_i + L + \cancel{2K} + P + -j_a + 2j_a' + j_s + j + j' + \cancel{\ }}$$

$w = \Omega - \Omega'$

$t = \Omega_i - \Omega$

$t' = \Omega_i' - \Omega'$

$$q + \boxed{\Omega - \Omega' - \Omega_i - \Omega_i'} \longrightarrow = -(\Omega_i - \Omega) - (\Omega_i' - \Omega') - 2\Omega'$$

$$= -t - t' + 2j'$$

$\cancel{q = 0}$

$q = -\Omega_0 + \Omega = -t'$

$q' = -\Omega_0' + \Omega' = -t$

$$\begin{pmatrix} 1 & 1 & P \\ q & -q' & q'-q \end{pmatrix} = \begin{pmatrix} 1 & 1 & P \\ -t' & t & t'-t \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 1 & P \\ -t & t' & t-t' \end{pmatrix}$$

$$\begin{pmatrix} J_0 & J_0' & K \\ \Omega_0 & -\Omega_0' & \Omega_0'-\Omega_0 \end{pmatrix} = (-\ \ \begin{pmatrix} J_0' & J_0 & K \\ -\Omega_0' & -\Omega_0 & +\Omega_0'+\Omega_0 \end{pmatrix}$$

$$\begin{pmatrix} S & L & K \\ p & -M_L & -Q \end{pmatrix} = \begin{pmatrix} K & L & S \\ Q & M_L & -p \end{pmatrix}$$

$$\begin{pmatrix} P & K & S \\ q-q' & \Omega_0-\Omega_0' & \Omega'-\Omega \end{pmatrix} = \begin{pmatrix} \ell & K_i & P \\ t-t' & \Omega_i'-\Omega_i & \Omega-\Omega' \end{pmatrix} = (-1)^{\ell+K_i+P} \begin{pmatrix} P & K_i & \ell \\ w & \Omega_i'-\Omega_i & t-t' \end{pmatrix}$$

$$\begin{pmatrix} P & K & S \\ 0 & -p & p \end{pmatrix} = \begin{pmatrix} \ell & K_i & P \\ 0 & -Q_i & Q_i \end{pmatrix} = (-1)^{\ell+K_i+P} \begin{pmatrix} \ell & K_i & P \\ 0 & Q_i & -Q_i \end{pmatrix}$$

$$\begin{pmatrix} S & L & K \\ \Omega-\Omega' & 0 & \Omega'-\Omega \end{pmatrix} = \begin{pmatrix} P & L & K \\ \Omega'-\Omega & 0 & \Omega-\Omega' \end{pmatrix} = \begin{pmatrix} K & L & P \\ \Omega'-\Omega & 0 & \Omega-\Omega' \end{pmatrix}$$

$$= \begin{pmatrix} k & L & p \\ -w & c & w \end{pmatrix}$$

$$\begin{pmatrix} S & L & K \\ \rho & -M_L & -Q \end{pmatrix} = \begin{pmatrix} K & L & P \\ Q & M_L & \cancel{-Q_i} \\ & & p \end{pmatrix}$$

$\rho = C$

$-Q_i$

$g = Q_i + p$

$p = -Q_i$

$$\begin{pmatrix} j' & j & k \\ \Omega' & -\Omega & \Omega-\Omega' \end{pmatrix} = \begin{pmatrix} j & j' & k \\ \Omega & -\Omega' & \Omega'-\Omega \end{pmatrix}$$

Phase $(-1)^{-J_0' + \rho + L + \underline{\underline{S}} + \rho \;+ 2j_a - j_a' + j_b + j + j'}$ $\quad 2g = 2\underline{K_i}$

$$(-1)^{p} (2\ell+1)^{1/2} \begin{pmatrix} 1 & 1 & \ell \\ p & -p & 0 \end{pmatrix} e_{-p}\, e^{*}_{-p}$$

$$= (-1)^{-p} [\ell]^{1/2} \begin{pmatrix} 1 & 1 & \ell \\ p & -p & 0 \end{pmatrix} e_{-p}\, e^{*}_{-p}$$

Gorril $\quad M_i + L + P \qquad + 2j_a' - j_a + j_b + j + j'$

$M_i + 2j' \qquad | \quad - M_i + \cancel{2K_i}$

$(-1)^{2(M_i + j')} = 1 \qquad \underline{OK}$