16 juli 2011

(1)

$$D^{(J),*}_{M,\Omega}(\alpha,\beta,\gamma)\ \hat{R}(\alpha,\beta,\gamma)\ |S\Sigma\Lambda\Omega\rangle \qquad \sqrt{\frac{2J+1}{4\pi}}$$

Multipole moments

$$\langle \Lambda\,\Omega\,JM|\ \hat{Q}_{kq}\ |\underbrace{\Lambda'\Omega'}\,J'M'\rangle$$

$$\hat{Q}_{kq} = \sum_i q_i\, R_{kq}(\hat{r}_i)$$

$$= \frac{[J]^{1/2}[J']^{1/2}}{4\pi}\ D^{(J)}_{M,\Omega}(\alpha\beta\gamma)\ \cancel{\hat{R}\hat{Q}_{kq}\hat{R}}\ D^{(J')}_{M'\Omega'}$$

$$\langle \Lambda\,\Omega|\ \hat{R}^{+}\hat{Q}_{kq}\hat{R}\ |\Lambda'\Omega'\rangle$$

$$\left[\ \hat{R}^{+}\hat{Q}_{kq}\hat{R} = \sum_{q'} \hat{Q}_{kq'}\ D^{(k)}_{q'q}(\hat{R}^{+})\right.$$

$$\left. = \sum_{q'} \hat{Q}_{kq'}\ D^{(k),*}_{q\,q'}(\alpha,\beta,\gamma)\ \right]$$

$$= \frac{[J,J']^{1/2}}{4\pi}\int_0^{2\pi}\!\!\int_{-1}^{1} D^{(J)}_{M,\Omega}(\alpha,\beta,\gamma)\ D^{(k)*}_{qq'}(\alpha,\beta,\gamma)\ D^{(J')}_{M,\Omega'}(\alpha,\beta,\gamma)$$

$$\langle \Lambda\,\Omega|\ \hat{Q}_{kq'}\ |\Lambda'\Omega'\rangle \qquad d\alpha\, d\cos\beta$$

$$\frac{1}{2\pi}\int_0^{2\pi} d\gamma = 1$$

(16 jul 2011) ②

$$= \frac{[J,J']^{1/2}}{8\pi} \iiint D^{(J)}_{M,\Omega}(\alpha\beta\gamma)\, D^{(k)*}_{qq'}(\alpha\beta\gamma)\, D^{(J')*}_{M',\Omega'}(\alpha\beta\gamma)\, d\alpha\, d\gamma\, d\cos\beta$$

$$\times \langle \Lambda\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle$$

$$= \frac{[J,J']^{1/2}}{[J]} \langle kq\, J'M' | JM \rangle \langle kq'\, J'\Omega' | J\Omega \rangle$$

$$\langle \Lambda\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle$$

$$= \sqrt{\frac{2J'+1}{2J+1}} \langle kq\, J'M' | JM \rangle \langle kq'\, J'\Omega' | J\Omega \rangle$$

$$\langle \Lambda\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle$$

$$\langle kq\, J'M' | JM \rangle = (-1)^{k-J'+M} [J]^{1/2} \begin{pmatrix} k & J' & J \\ q & M' & -M \end{pmatrix}$$

$$\langle kq'\, J'\Omega' | J\Omega \rangle = (-1)^{k-J'+\Omega} [J]^{1/2} \begin{pmatrix} k & J' & J \\ q' & \Omega' & -\Omega \end{pmatrix}$$

$$\Downarrow$$

$$(-1)^{M-\Omega}$$

$$= [J,J']^{1/2} \begin{pmatrix} J & k & J' \\ -M & q & M' \end{pmatrix} \begin{pmatrix} J & k & J' \\ -\Omega & q' & \Omega' \end{pmatrix} (-1)^{M-\Omega}$$

$$\times \langle \Lambda\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle$$

$$\begin{array}{c} 1 - \\ 2 - \\ 3 - \\ 4 - \end{array} \begin{bmatrix} -3/2 \\ -1/2 \\ 1/2 \\ 3/2 \end{bmatrix}$$

16 juli 2011

$$(-1)^{M-\Omega} = (-1)^{J-\Omega}(-1)^{M-J} = (-1)^{J-\Omega}(-1)^{J-M}$$

$$= (-1)^{J-M}[k]^{1/2}\begin{pmatrix} J & k & J' \\ -M & q & M' \end{pmatrix}$$

$$\times (-1)^{J-\Omega}[k]^{1/2}\begin{pmatrix} J & k & J' \\ -\Omega & q' & \Omega' \end{pmatrix}$$

$$\times \frac{[J,J']^{1/2}}{[k]} \langle \Lambda\,\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle$$

ja / jh

$$\hat{Q}_{kq}(\Lambda\,\Omega\,J | \Lambda'\Omega'J') = \frac{[J,J']^{1/2}}{[k]} \hat{T}_{kq}(J;J')$$

$$\times \sum_{q'} \langle \Lambda\,\Omega | \hat{Q}_{kq'} | \Lambda'\Omega' \rangle (-1)^{J-\Omega}[k]^{1/2}\begin{pmatrix} J & k & J' \\ -\Omega & q' & \Omega' \end{pmatrix}$$

$$\langle J M | \hat{Q}_{kq} | J'M' \rangle \qquad \langle J M | T_{kq} | J'M' \rangle$$

$$|||$$

$$v^{(JJ')}_{kq;MM'}$$

$OH(^2\Pi_{\Omega_a}; j_a m_a) \quad + \quad NO(^2\Pi_{\Omega_b}; j_b m_b)$

16 jul 2011

molecular basis:

$$|S_a \Lambda_a \Omega_a; j_a m_a\rangle = \frac{[j_a]}{\sqrt{4\pi}} D^{(j_a)*}_{m_a \Omega_a}(\alpha_a, \beta_a, 0) \,|\, \hat{R}(\alpha_a \beta_a 0)\, |S_a \Lambda_a \Omega_a\rangle$$

coupled space-fixed basis

$$|\{[(S_a \Lambda_a \Omega_a)\, j_a\, (S_b \Lambda_b \Omega_b)\, j_b]\, j_{ab}\, l\}\, J M\rangle$$

$$= \sum_{m_a m_b m_{ab}} |S_a \Lambda_a \Omega_a; j_a m_a\rangle\, |S_b \Lambda_b \Omega_b; j_b m_b\rangle\, |l m\rangle \langle j_a m_a\, j_b m_b | j_{ab} m_{ab}\rangle \langle j_{ab} m_{ab}\, l m | J M\rangle$$

multipole operators $\quad \underline{r}_i^{(a)}: \; r_i^{(a)} = |\underline{r}_i^{(a)}| \; ; \; \hat{r}_i^{(a)} = \dfrac{\underline{r}_i^{(a)}}{r_i^{a}}$

$$\hat{Q}_{k_a q_a} = \sum_{i \in a} q_i^{(a)} \left[r_i^{(a)}\right]^{k_a} C_{k_a q_a}(\hat{r}_i^{(a)})$$

16 jul 2011

Tensor part of the problem

interaction operator

$$\left[\left[\hat{T}_{k_a q_a}(\Omega_a j_a ; \Omega_a' j_a') \otimes \hat{T}_{k_b}(\Omega_b j_b ; \Omega_b' j_b') \right]^{(k)} \otimes \hat{T}_k(l, l') \right]_0^{(0)}$$

$$= \sum_{j_{ab} j_{ab}'} \hat{T}^{*}_{kq}[(\Omega_a j_a \Omega_b j_b) j_{ab} ; (\Omega_a' j_a' \Omega_b' j_b') j_{ab}'] \begin{Bmatrix} j_a & j_a' & k_a \\ j_b & j_b' & k_b \\ j_{ab} & j_{ab}' & k \end{Bmatrix}$$

$$\times [j_{ab}, j_{ab}', k_a, k_b]^{1/2}$$

$$\left[\hat{T}_k(j_{ab} ; j_{ab}') \otimes \hat{T}_k(l ; l') \right]_0^{(0)} = \sum_J \hat{T}_{00}[(j_{ab} l) J ; (j_{ab}' l') J^*]$$

$$\times \begin{Bmatrix} j_{ab} & j_{ab}' & k \\ l' & l & J \end{Bmatrix} [J, k]^{1/2} (-1)^{j_{ab}' + l + k + J}$$

---

full answer:

$$\sum_{j_{ab} j_{ab}' J} \hat{T}_{00}[(\Omega_a j_a \Omega_b j_b) j_{ab} l J ; (\Omega_a' j_a' \Omega_b') j_{ab}' l' J]$$

$$[j_{ab}, j_{ab}', k_a, k_b, k, J]^{1/2} (-1)^{j_{ab}' + l + k + J}$$

$$\begin{Bmatrix} j_{ab} & j_{ab}' & k \\ l' & l & J \end{Bmatrix} \begin{Bmatrix} j_a & j_a' & k_a \\ j_b & j_b' & k_b \\ j_{ab} & j_{ab}' & k \end{Bmatrix}$$

$$\sum_{j_{ab} j_{ab}' J} \left(\ldots \right)^2 = 1$$

for fixed $j_a j_b' j_a' j_b\ l'$ and k_a, k_b, k

16 jul 2011

parity

$$\hat{i}\,|s_a\,\lambda_a\,\Omega_a\,j_a\,m_a\rangle = (-1)^{j_a - s_a}\,|s_a, -\lambda_a, -\Omega_a\,j_a\,m_a\rangle$$

with 1-electron C&S:

$$\hat{\sigma}_v(xz)\,|\lambda_a\rangle = |-\lambda_a\rangle \qquad [\hat{\sigma}_v(xz)\,Y_{lm}(\theta,\phi) = (-1)^m\,Y_{l,-m}(\theta,\phi)]$$

drop $s_a\,\lambda_a$ in notation

$$\hat{i}\,|\{(s_a\lambda_a\Omega_a)\,j_a\,(s_b\lambda_b\Omega_b)\,j_b\;j_{ab}\;l\};JM\rangle \qquad \hat{i}\,|lm\rangle = (-1)^l\,|lm\rangle$$

$$= (-1)^{j_a + j_b - s_a - s_b + l}\,|\{(s_a\,-\lambda_a\,-\Omega_a)\,j_a\,(s_b\,-\lambda_b\,-\Omega_b)\,j_b\;j_{ab}\;l\};JM\rangle$$

counting channels

$j_a = 3/2,\ 5/2,\ 7/2 \qquad \Omega = 3/2 \qquad \pm \quad \rightarrow \quad \cancel{2 \times 3 = 6}$

$j_b = 1/2,\ 3/2,\ 5/2 \qquad \Omega = 1/2, \quad \pm \quad \rightarrow$

$\mathcal{H}_2$

parity adapted monomer

$$|s_a\lambda_a\Omega_a;\,j_a m_a;\,p_a\rangle = \left[\frac{1 + p_a\,\hat{i}_a}{\sqrt{2}}\right]|s_a\,\lambda_a\,\Omega_a\,j_a\,m_a\rangle$$

$$|s_b\,\lambda_b\,\Omega_b;\,j_b m_b;\,p_b\rangle = \frac{1 + p_b\,\hat{i}_b}{\sqrt{2}}\,|s_b\,\lambda_b\,\Omega_b\,j_b\,m_b\rangle$$

Total parity $\quad p_a\,p_b\,(-1)^l$

$$|(s_a\lambda_a\Omega_a)\,j_a\,(s_b\lambda_b\Omega_b)\,j_b\;j_{ab}\;l\;p_a p_b;JM\rangle$$

$$= \underbrace{\frac{1 + p_a\,\hat{i}_a}{\sqrt{2}}}_{\hat{P}_a}\;\underbrace{\frac{1 + p_b\,\hat{i}_b}{\sqrt{2}}}_{\hat{P}_b}\;|(s_a\lambda_a\Omega_a)\,j_a\,(s_b\lambda_b\Omega_b)\,j_b\;j_{ab}\;l;JM\rangle$$

continue page 10

Multipole operator.

$$\hat{Q}^{(a)}_{k_a q_a}(\Omega_a j_a; \Omega_a' j_a') = \frac{[j_a, j_a']^{1/2}}{[k_a]} \hat{T}_{k_a q_a}(\Omega_a j_a; \Omega_a' j_a')$$

$$\times \sum_{q_a'} \langle \Omega_a | \hat{Q}^{(a)}_{k_a q_a'} | \Omega_a' \rangle (-1)^{j_a - \Omega_a} [k_a]^{1/2} \begin{pmatrix} j_a & k_a & j_a' \\ -\Omega_a & q_a' & \Omega_a' \end{pmatrix}$$

$$\hat{Q}^{(b)}_{k_b q_b}(\Omega_b j_b; \Omega_b' j_b') = \frac{[j_b, j_b']^{1/2}}{[k_b]} \hat{T}_{k_b q_b}(\Omega_b j_b; \Omega_b' j_b')$$

$$\times \sum_{q_b'} \langle \Omega_b | \hat{Q}^{(b)}_{k_b q_b'} | \Omega_b' \rangle (-1)^{j_b - \Omega_b} [k_b]^{1/2} \begin{pmatrix} j_b & k_b & j_b' \\ -\Omega_b & q_b' & \Omega_b' \end{pmatrix}$$

$$\sum \hat{T}_{kq} \times \hat{T}_{k,-q} (-1)^q$$

$$\sum_{q q'} T_{kq} T_{kq'} \langle kq\, kq' | 00 \rangle$$

$$= \sum_q T_{kq} T_{k,-q} \frac{(-1)^{k-q}}{\sqrt{2k+1}}$$

$$\boxed{= \sum_q \hat{T}_{kq} \hat{T}_{k,-q} (-1)^q = [k]^{1/2} (-1)^k \left[\hat{T}_k \otimes \hat{T}_k\right]^{(0)}_0}$$

$$\hat V = \sum_k \frac{C^*_{kq}(\hat R)}{R^{k+1}} \sum_{k_1+k_2=k} \sum_{q_a+q_b=q} (-1)^{k_2} \binom{2k}{2k_1}^{1/2} \left[\hat Q_{k_a q_a} \otimes \hat Q_{k_b q_b}\right]^{(k)}_q$$

$$= \sum_{k_1 k_2} \frac{1}{R^{k_1+k_2+1}} (-1)^{k+k_2} [k]^{1/2} \binom{2k}{2k_1}^{1/2} \left[\left[\hat Q_{k_a} \otimes \hat Q_{k_b}\right]^{(k)}_{\cdot} \times C_{k\cdot}\right]^{(0)}_0$$

$$k = k_1 + k_2 \Rightarrow k + k_2 = \cancel{2k_1} + k_1 + 2k_2$$

$$(-1)^{k+k_2} = (-1)^{k_1}$$

$$\hat V = \sum_{k_a k_b} \frac{1}{R^{k_a+k_b+1}} (-1)^{k_a} [k]^{1/2} \binom{2k}{2k_a}^{1/2} \left[\left[\hat Q_{k_a} \otimes \hat Q_{k_b}\right]^{(k)} \otimes C_{k\cdot}\right]^{(0)}_0$$

$$C_{kq}(\theta,\phi) = \sum_{\ell \ell'} \hat T_{kq}(\ell;\ell')\, c_k(\ell,\ell')$$

$$c_k(\ell,\ell') = (-1)^{\ell} \frac{[\ell,\ell']^{1/2}}{[k]^{1/2}} \begin{pmatrix} \ell & k & \ell' \\ 0 & 0 & 0 \end{pmatrix}$$

(9)

$$k = k_a + k_b$$

$$\hat{V} = \sum_{k_a k_b} \frac{1}{R^{k_a+k_b+1}} (-1)^{k_a} [k]^{1/2} \binom{2k}{2k_a}^{1/2}$$

$$\times \frac{[j_a, j_a', j_b, j_b']^{1/2}}{[k_a, k_b]^{1/2}} (-1)^{j_a - \Omega_a + j_b - \Omega_b}$$

$$\begin{pmatrix} j_a & k_a & j_a' \\ -\Omega_a & q_a' & \Omega_a' \end{pmatrix} \begin{pmatrix} j_b & k_b & j_b' \\ -\Omega_b & q_b' & \Omega_b' \end{pmatrix}$$

$$\langle \Omega_a | \hat{Q}_{k_a q_a'} | \Omega_a' \rangle \langle \Omega_b | \hat{Q}_{k_b q_b'} | \Omega_b' \rangle$$

$$(-1)^{l} \frac{[l, l']^{1/2}}{[k]^{1/2}} \begin{pmatrix} l & k & l' \\ 0 & 0 & 0 \end{pmatrix}$$

$$\left[\left[T_{k_a}(\Omega_a j_a; \Omega_a' j_a') \otimes T_{k_b}(\Omega_b j_b; \Omega_b' j_b') \right]^{(k)} \otimes \hat{T}_k(l, l') \right]^{(0)}_0$$

$$\hat{P}_a = \frac{1 + p_a \hat{i}_a}{\sqrt{2}}$$

$$\hat{P}_a^2 = \frac{(1+p_a\hat{i}_a)(1+p_a\hat{i}_a)}{2} = \frac{1 + p_a^2 \hat{i}_a^2 + 2p_a \hat{i}_a}{2}$$

$$= 1 + p_a \hat{i}_a$$

$$\hat{P}_b^2 = 1 + p_b \hat{i}_b \qquad \hat{P}_a^+ = \hat{P}_a$$

$$\langle \hat{P}_a \hat{P}_b \qquad \hat{i}$$

$$\hat{i}_l \, | \, l m\rangle = (-1)^l \, | \, l m \rangle$$

$$[(\hat{i}_a \hat{i}_b \hat{i}_l), \hat{V}] = 0 \qquad (-1)^l p_a p_b = P$$

$$(|\mathcal{r}_a\rangle + p_a|-\mathcal{r}_a\rangle)\,(|\mathcal{r}_b\rangle + p_b|-\mathcal{r}_b\rangle)$$

$$= |\mathcal{r}_a \mathcal{r}_b\rangle + p_b|\mathcal{r}_a - \mathcal{r}_b\rangle + p_a|-\mathcal{r}_a \mathcal{r}_b\rangle + p_a p_b|-\mathcal{r}_a - \mathcal{r}_b\rangle$$

$$= |\mathcal{r}_a \mathcal{r}_b l\rangle + \overline{p_a p_b}\,|-\mathcal{r}_a - \mathcal{r}_b l\rangle$$

$$+ p_b|\mathcal{r}_a - \mathcal{r}_b l\rangle + p_a|-\mathcal{r}_a \mathcal{r}_b l\rangle$$

$$\frac{1+P\hat{i}}{\sqrt{2}} |\lambda_a \lambda_b \ell\rangle =$$

$$|\lambda_a \, {-\lambda_b} \, \ell\rangle$$

$$\frac{(1+P\hat{i})}{\sqrt{2}} |j_a \lambda_a j_b \lambda_b \; s_a s_b \; i_a i_b \; j_{ab}\rangle \ell$$

$$\frac{1+P\hat{i}}{\sqrt{2}} |\lambda_a \lambda_b \ell\rangle = \frac{1}{\sqrt{2}} |\lambda_a \lambda_b \ell\rangle + \frac{P(-1)^{j_a - s_a + i_b - s_b + \ell}}{\sqrt{2}} |{-\lambda_a} {-\lambda_b} \ell\rangle$$

$$\frac{1+P\hat{i}}{\sqrt{2}} |\lambda_a {-\lambda_b} \ell\rangle = \frac{1}{\sqrt{2}} |\lambda_a {-\lambda_b} \ell\rangle + \frac{P(-1)^{j_a - s_a + j_b - s_b + \ell}}{\sqrt{2}} |{-\lambda_a} \lambda_b \ell\rangle$$

convention : $\lambda_a > 0$

$$\hat{i} = \hat{i}_a \hat{i}_b \hat{i}_\ell \qquad \hat{i}_\ell |\ell\rangle = (-1)^\ell |\ell\rangle$$

$$\hat{i} = P_{tot} \qquad \hat{i}_a = \hat{i} \, \hat{i}_b \, \hat{i}_\ell$$

$$|\lambda_a P_a\rangle |\lambda_b P_b\rangle |\ell\rangle$$

$$P_a P_b \ell = P_{tot}$$

$$\langle \lambda_a P_a \lambda_b P_b \ell | \hat{V} | \lambda_a' P_a' \lambda_b P_b' \ell'\rangle$$

$$= \left\langle \frac{1+P_a \hat{i}_a}{\sqrt{2}} \frac{1+P_b \hat{i}_b}{\sqrt{2}} |\lambda_a \lambda_b \ell \right| \frac{1+P_a' \hat{i}_a}{\sqrt{2}} \frac{1+P_b'}{\sqrt{2}} |\lambda_a \lambda_b \ell'\rangle$$

12

$$\hat{r}_a = \hat{\imath}_b \hat{\imath}_\ell \hat{\imath} \qquad p_a \hat{\imath}_b +$$

$$\tfrac{1}{2} (1 + p_a \hat{\imath}_b \hat{\imath}_\ell \hat{\imath}) (1 + p_b \hat{\imath}_b)$$

$$= \tfrac{1}{2} [1 + p_a \underline{\hat{\imath}_b \hat{\imath}_\ell} \hat{\imath} + p_b \hat{\imath}_b + p_a p_b \underline{\hat{\imath}_\ell} \hat{\imath}]$$

$$= \tfrac{1}{2} [(1 + p_b \hat{\imath}_b) + p_a (\hat{\imath}_b + p_b \hat{\imath}_b) \hat{\imath}_\ell \hat{\imath}]$$

$$= \tfrac{1}{2} [(1 + p_b \hat{\imath}_b)$$

$$= \tfrac{1}{2} [1 + p_b \hat{\imath}_b + p_a \hat{\imath}_b \hat{\imath}_\ell \hat{\imath} + p_a \cancel{p_b} \hat{\imath}_\ell \hat{\imath}]$$

$$= \tfrac{1}{2} [1 + p_b \hat{\imath}_b + p_a (\underline{\hat{\imath}_b + p_b}) \hat{\imath}_\ell \hat{\imath}]$$

$$p_b (p_b \hat{\imath}_b + 1)$$

$$= p_b (1 + p_b \hat{\imath}_b)$$

$$= \tfrac{1}{2} [1 + p_b \hat{\imath}_b + p_a p_b (1 + p_b \hat{\imath}_b) \hat{\imath}_\ell \hat{\imath}]$$

$$= \tfrac{1}{2} (1 + p_b \hat{\imath}_b) (1 + p_a p_b \overset{(-1)^\ell}{\hat{\imath}_\ell} \hat{\imath})$$

$$= \cancel{(1 + p_a p_b (-1)^\ell \hat{\imath}) (1 + p_a' p_b' (-1)^{\ell'} \hat{\imath})}$$

$$p_a p_b (-1)^\ell = p_{tot}$$

$$= \tfrac{1}{2} (1 + p_b \hat{\imath}_b) (1 + \underline{p_{tot}} \hat{\imath})$$
$$ 2$$

$$\langle p_a p_b l | \hat{V} | p_a' p_b' l' \rangle$$

$$= \left\langle \frac{1+P_{tot}\hat{\imath}}{\sqrt{2}} \left(\frac{1+P_b\hat{\imath}_b}{\sqrt{2}}\right) \right| \hat{V} \left| \frac{1+P_{tot}\hat{\imath}}{\sqrt{2}} \left(\frac{1+P_b'\hat{\imath}_b}{\sqrt{2}}\right) s_a' s_b' l' \right\rangle$$

$$\left[\frac{1+P_{tot}\hat{\imath}}{\sqrt{2}}\right]^2 = \frac{(1+P_{tot}\hat{\imath})^2}{2} = 1+P_{tot}\hat{\imath}$$

$$= \left\langle (1+P_b\hat{\imath}_b)^{\dagger} [s_a s_b l] \right| \hat{V} \, (1+P_{tot}\hat{\imath}) \left(\frac{1+P_b'\hat{\imath}_b}{2}\right) \left| s_a' s_b' l' \right\rangle$$

$$= \frac{1}{2} \left\langle (1+P_b\hat{\imath}_b) | s_a s_b l \right| \hat{V} \left| (1+P_{tot}\hat{\imath}) \right. (1+P_b'\hat{\imath}_b) \left| s_a' s_b' l' \right\rangle$$

$$(1+P_{tot}\hat{\imath})(1+P_b'\hat{\imath}_b)$$

$$= \left[1 + P_a' P_b' (-1)^{l'} \hat{\imath}_a \hat{\imath}_b \hat{\imath}_l\right] \left[1+P_b'\hat{\imath}_b\right]$$

$$(-1)^{l'}$$

$$= \left[1 + P_a' P_b' \hat{\imath}_a \hat{\imath}_b\right] \left[1+P_b'\hat{\imath}_b\right]$$

$$= \left[1 + P_a' P_b' \hat{\imath}_a \hat{\imath}_b + P_b'\hat{\imath}_b + P_a'\hat{\imath}_a\right]$$

$$= (1+P_a'\hat{\imath}_a)(1+P_b'\hat{\imath}_b)$$

$$= \frac{1}{2}\langle \underbrace{(1+\rho_b\tau_b)}_{2}\, \lambda_a \lambda_b\, \ell \,|\, V\, \underbrace{(1+\rho_a'\tau_a)(1+\rho_b'\tau_b)}_{2\times 2}\, \lambda_a'\lambda_b'\, \ell \rangle \qquad = 8$$

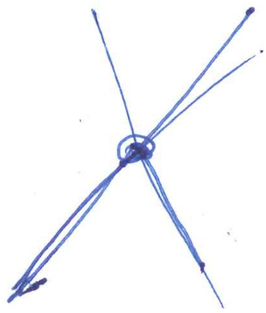
$\langle 1\ 1 \mid 1\ 1\rangle$ $\qquad$ $\langle 1\ {-1} \mid 1\ 1\rangle$

$\langle 1\ 1 \mid 1\ {-1}\rangle$ $\qquad$ $\langle 1\ {-1} \mid 1\ {-1}\rangle$

$\langle 1\ 1 \mid {-1}\ 1\rangle$ $\qquad$ $\langle 1\ {-1} \mid {-1}\ 1\rangle$

$\langle 1\ 1 \mid {-1}\ {-1}\rangle$ $\qquad$ $\langle 1\ {-1} \mid {-1}\ {-1}\rangle \to 4\pi$

$3 \cdot \bar{\xi}$



$$e^{i\phi(\xi)}$$

$$\pm\sqrt{\xi_1^2+\xi_2^2}$$

$$\int |\phi_x(x)|^2 |\phi_y(y)|^2 e^{i\sqrt{x^2+y^2}}$$