Polarized light

Direction of photon: $\hat{k}$, $|\hat{k}| \equiv 1$, real vector

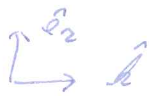
Linear polarization $\hat{e}_z \perp \hat{k}$

Define $\hat{e}_x = \hat{k} \times \hat{e}_z$

General elliptically polarized light:

$$\hat{e} = \cos\beta \cdot \hat{e}_z + i \sin\beta \, \hat{e}_x$$

$\beta = 0$



Linearly polarized

$\beta = \frac{\pi}{4}$ $\quad \hat{e} = \frac{1}{2}\sqrt{2}\,[\hat{e}_z + i\,\hat{e}_x]$ right circularly polarized
travelling direction $\hat{k} = \hat{e}_z \times \hat{e}_x$

$\beta = -\frac{\pi}{4}$ $\quad \hat{e} = \frac{1}{2}\sqrt{2}\,[\hat{e}_z - i\,\hat{e}_x]$ Left circularly polarized

Alternatively arbitrary complex vector $\hat{e}$

travel direction: $Re(\hat{e}) \times Im(\hat{e}) = \hat{k}$

Linear polarization: $Re(\hat{e})$

$$\hat{e} = e_x \hat{e}_x + e_y \hat{e}_y + e_z \hat{e}_z$$

Spherical components

$$e_0 = e_z$$

$$e_{\pm 1} = \mp \frac{e_x \pm i e_y}{\sqrt{2}}$$

Interaction operator $\hat{A} = \hat{e} \cdot \mu = \sum_m (-1)^m e_{-m} \mu_m$

Dipole operator

$$\mu_m(r_1 \ldots r_n) = \sum_{i=1}^{n} q_i R_{1,m}(r_i) \qquad R_{l,m}(r) = r^l C_{lm}(\hat{r})$$

Components of $e^* = (e_x^*, e_y^*, e_z^*)$

$$(e^*)_0 = (e^*)_z = e_z^* = e_0^*$$

$$(e^*)_{\pm 1} = \mp \frac{e_x^* \pm i e_y^*}{\sqrt{2}} = \mp \left[\frac{e_x \mp i e_y}{\sqrt{2}}\right]^* = -\left[\pm \frac{e_x \mp i e_y}{\sqrt{2}}\right]^* = -[e_{\mp}]^*$$

$$(e^*)_m = (-1)^m [e_{-m}]^* = (-1)^m e_{-m}^*$$

One photon absorption

Single particle, SF description

$$\psi_{l_1 m_1}(\underline{r}) = \frac{f_{l_1}(r)}{r} Y_{l_1 m_1}(\hat{r})$$

transition dipole matrix

$$\langle \psi_{l_1 m_1} | \mu_m | \psi_{l_2 m_2} \rangle = \iint Y^*_{l_1 m_1}(\hat{r})\, C_{1m}(\hat{r})\, Y_{l_2 m_2}(\hat{r})\, d\hat{r}$$

$$\times \int \frac{f^*_{l_1}(r)}{r}\, q \cdot r\, \frac{f_{l_2}(r)}{r}\, r^2 dr$$

Angular part

$$\iint Y^*_{l_1 m_1}(\hat{r})\, C_{1m}(\hat{r})\, Y_{l_2 m_2}(\hat{r})\, d\hat{r} = \frac{[l_1]^{1/2}[l_2]^{1/2}}{4\pi} \underbrace{\iint Y^*_{l_1 m_1}(\hat{r})\, C_{l_1 m_1}(\hat{r})\, d\hat{r}}_{= \frac{4\pi}{2l_1+1}} \langle 1m\, l_2 m_2 | l_1 m_1 \rangle \langle 10\, l_2 0 | l_1 0 \rangle$$

$$= [l_1]^{1/2}[l_2]^{1/2} (-1)^{1-l_2+m_1} \begin{pmatrix} 1 & l_2 & l_1 \\ m & m_2 & -m_1 \end{pmatrix} \begin{pmatrix} 1 & l_2 & l_1 \\ 0 & 0 & 0 \end{pmatrix} (-1)^{1-l_2}$$

$$= \underbrace{(-1)^{l_1 - m_1} \begin{pmatrix} l_1 & 1 & l_2 \\ -m_1 & m & m_2 \end{pmatrix} [3]^{1/2}}\, [l_1]^{1/2} [l_2]^{1/2} \left[\tfrac{1}{3}\right]^{1/2} \begin{pmatrix} l_1 & 1 & l_2 \\ 0 & 0 & 0 \end{pmatrix} (-1)^{l_1 + 2 - 2l_2}$$

$$\hat{\mu}_m = \sum_{l_1 l_2} \hat{T}_{1m}(l_1 l_2)\, a_{l_1 l_2}(r)$$

$$a_{l_1 l_2}(r) = q \cdot r \frac{[l_1]^{1/2}[l_2]^{1/2}}{[3]^{1/2}} (-1)^{l_1} \begin{pmatrix} l_1 & 1 & l_2 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\langle l_1 m_1 | \mu_m | l_2 m_2 \rangle = \langle \overline{a}_{l_1 l_2}(r) \rangle (-1)^{l_1 - m_1} \begin{pmatrix} l_1 & 1 & l_2 \\ -m_1 & m & m_2 \end{pmatrix} \sqrt{3}$$

27 dec 1999 (3)

$$\sum_k \left[[\underline{a}\times\underline{c}]^{(k)} \times [\underline{b}\times\underline{d}]^{(k)} \right]^{(0)}_0 [k]^{1/2}$$

$$= \sum_{\substack{k q_1 q_2 \\ m_1 m_2 m_3 m_4}} a_{m_1} b_{m_2} c_{m_3} d_{m_4} \langle 1 m_1 1 m_3 | k q_1 \rangle \langle 1 m_2 1 m_4 | k q_2 \rangle \langle k q_1 k q_2 | 0 0 \rangle [k]^{1/2}$$

$$= \sum_{\substack{k q_1 q_2 \\ m_1 m_2 m_3 m_4}} a_{m_1} b_{m_2} c_{m_3} d_{m_4} (-1)^{1-1+q_1} [k]^{1/2} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_3 & -q_1 \end{pmatrix} (-1)^{1-1+q_2} [k]^{1/2} \begin{pmatrix} 1 & 1 & k \\ m_2 & m_4 & -q_2 \end{pmatrix} (-1)^{k-q_1} \delta_{q_1,-q_2}$$

$$= \sum_{\substack{k q_1 \\ m_1 m_2 m_3 m_4}} a_{m_1} b_{m_2} c_{m_3} d_{m_4} (-1)^{m_2+m_4} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_3 & -q_1 \end{pmatrix} \underbrace{\begin{pmatrix} 1 & 1 & k \\ m_2 & m_4 & +q_1 \end{pmatrix} (-1)^{k}}_{= \begin{pmatrix} 1 & 1 & k \\ -m_2 & -m_4 & -q_1 \end{pmatrix}} [k]$$

$$\sum_{m_1 m_2 m_3 m_4} (-1)^{m_2} a_{m_1} b_{m_2} \delta_{m_1,-m_2} \quad (-1)^{m_4} c_{m_3} d_{m_4} \delta_{m_3,-m_4}$$

$$= (\underline{a}\cdot\underline{b})(\underline{c}\cdot\underline{d})$$

1-photon process

$$\sigma_{j_1 m_1; j_2 m_2} = C \sum_i \langle j_1 m_1 | \hat{e}\cdot\mu | i \rangle p_i \langle i | \hat{e}^*\cdot\mu | j_2 m_2 \rangle$$

$$= C \sum_k \sum_{m_1 m_3} e_{m_1} (e^*)_{m_3} \langle 1 m_1 1 m_3 | k q_1 \rangle \sum_{m_2' m_4'} \sum_i \langle j_1 m_1 | \mu_{m_2'} | i \rangle p_i \langle i | \mu_{m_4'} | j_2 m_2 \rangle$$

$$\times \langle 1 m_2' 1 m_4' | k q_2 \rangle \langle k q_1 k q_2 | 0 0 \rangle [k]^{1/2}$$

$$E_{kq}(\underline{e}) \equiv \sum_{m_1 m_3} e_{m_1} (e^*)_{m_3} \langle 1 m_1 1 m_3 | k q \rangle = \sum_{m_1 m_3} e_{m_1} (-1)^{m_3} e^*_{-m_3} (-1)^{1-1+q} [k]^{1/2} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_3 & -q \end{pmatrix}$$

$$q = m_1 + m_3 \qquad 2m_3 + m_1 = m_1$$

$$E_{kq}(\underline{e}) = \sum_{m_1 m_3} (-1)^{m_1} e_{m_1} e^*_{-m_3} \begin{pmatrix} 1 & 1 & k \\ m_1 & m_3 & -q \end{pmatrix} [k]^{1/2}$$

$$= [k]^{1/2} \sum_{m_1 m_3} (-1)^{m_1} e_{m_1} e^*_{m_3} \begin{pmatrix} 1 & 1 & k \\ m_1 & -m_3 & -q \end{pmatrix}$$

$$\sigma_{f'f} = c \sum_i \langle f'_{R_1} | \hat{e}_{R_1} \cdot \mu | i_{R_1} \rangle p_i \langle i_{R_2} | \hat{e}^*_{R_2} \cdot \mu | f_{R_2} \rangle$$

$$= c \sum_g \langle f'_1 | \hat{e}_1 \cdot \mu \, \hat{\rho} | g \rangle \langle g | \hat{e}^*_2 \cdot \mu | f_2 \rangle$$

$$= c \sum_g \langle g | \hat{e}^*_2 \cdot \mu | f_2 \rangle \langle f'_1 | \hat{e}_1 \cdot \mu \, \hat{\rho}_1 | g \rangle$$

$$= c \; \mathrm{tr}\,(A^\dagger \hat{\rho})$$

$$\hat{A}^\dagger = \hat{e}^*_2 \cdot \mu \, | f_2 \rangle \langle f'_1 | \hat{e}_1 \cdot \mu$$

$$\hat{A} = \hat{e}^*_1 \cdot \mu \, | f'_1 \rangle \langle f_2 | \hat{e}^*_2 \cdot \mu = \overset{a}{\hat{e}^*} \cdot \overset{b}{\mu} \, \hat{P}_{f'f} \, \overset{c}{\hat{e}^*} \cdot \overset{d}{\mu}$$

$$\hat{A} = \sum_{\substack{k q_1 q_2 \\ m_1 m_2}} \underbrace{E^*_{k q_1}(\hat{e})} \; \underbrace{\mu_{m_1} \vec{P}_{f'f} \mu_{m_2} \langle 1 m_1 1 m_2 | k q_2 \rangle} \langle k q_1 k q_2 | 00 \rangle [k]^{1/2}$$

$$\sum_k \left[[\underline{a} \times \underline{c}]^{(k)}_{q_1} \quad [\underline{b} \times \underline{d}]^{(k)}_{q_1} \right]^{(0)}_0 [k]^{1/2}$$