Summary $\qquad [j]^{1/2} = \sqrt{2j+1}$ $\qquad$ 1

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$$j_1 m_1 \;\text{———}\; j_2 m_2 = \delta_{j_1 j_2}\,\delta_{m_1 m_2}$$

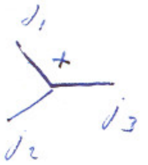
$$j_1 m_1 \;\longleftarrow\; j_2 m_2 = \delta_{j_1 j_2}\,\delta_{m_1,-m_2}\,(-1)^{j_1+m_1}$$

$$j\,{-m} \;\longrightarrow\; j\,m = (-1)^{j+m}$$

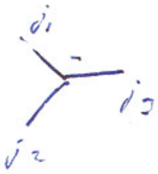
$$\rightarrow\rightarrow \;=\; (-1)^{2j}\;\text{——}$$

$$\rightarrow\leftarrow \qquad j m \rightarrow\leftarrow j m = (-1)^{2(j-m)}\;\text{——}$$

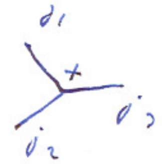
$$\rightarrow\rightarrow\rightarrow \;=\; \leftarrow$$



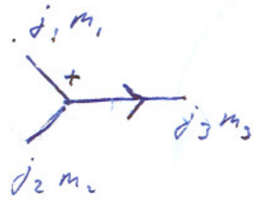
$$= \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$



$$= \begin{pmatrix} j_1 & j_3 & j_2 \\ m_1 & m_3 & m_2 \end{pmatrix} = (-1)^{j_1+j_2+j_3}$$



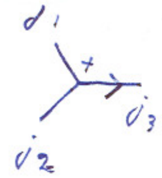
(ii)



$$= \sum_m \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m \end{pmatrix} \delta_{m,-m_3}\,(-1)^{j_3+m_3}$$

$$= \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix} (-1)^{j_3+m_3}$$

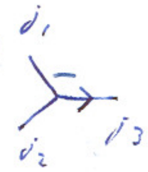
$(-1)^{j_1-j_2-j_3}\,[j_3]^{1/2}$



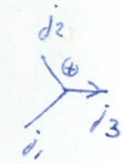
$$= (-1)^{j_1-j_2+m_3}\,[j_3]^{1/2} \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & -m_3 \end{pmatrix}$$

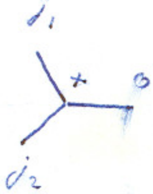
$$= \langle j_1 m_1 j_2 m_2 | j_3 m_3 \rangle$$

$(-1)^{2j_1}\,[j_3]^{1/2}$



$$= \langle j_1 m_1 j_2 m_2 | j_3 m_3 \rangle = (-1)^{2j_1}\,[j_3]^{1/2}$$





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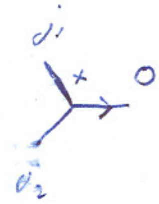
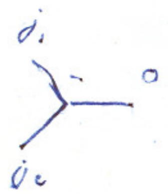


$$= \langle j_1 m_1 j_2 m_2 | 0 0 \rangle$$

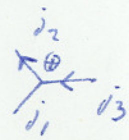
$$= \begin{pmatrix} j_1 & j_2 & 0 \\ m_1 & m_2 & 0 \end{pmatrix} = \delta_{j_1 j_2}\,\delta_{m_1,-m_2}\,(-1)^{j_1-m_1}\,[j_1]^{-1/2}$$

$$= \frac{1}{[j_1]^{1/2}}$$



$$= \frac{1}{[j_1]^{1/2}}$$

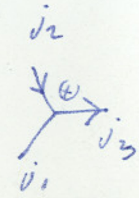
$[j_3]^{1/2}$



2.0.2

$\|$

$$\langle j_1 m_1 j_2 \bar{m}_2 | j_3 m_3 \rangle \equiv (-1)^{j_2-m_2} \langle j_1 m_1 j_2 {-m_2} | j_3 m_3 \rangle = (-1)^{2j_1}\,[j_3]^{1/2}$$



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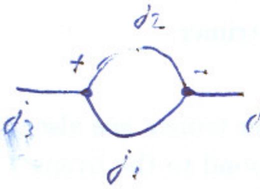
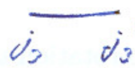
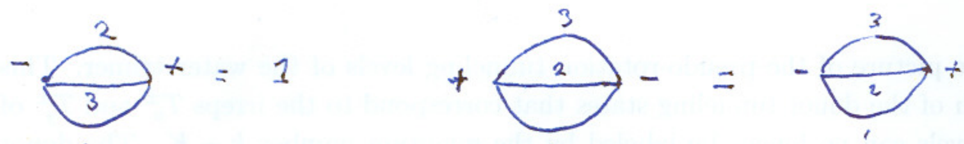
$= 2j+1 \qquad \sum_{m=-j}^{j} \delta_{m,m} = 2j+1$



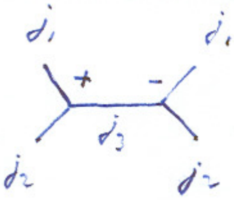
$= \frac{1}{2j_3+1}$



$\Delta(j_1 j_2 j_3)$

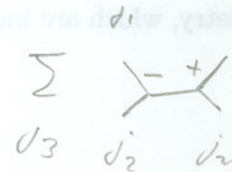


$\sum_{j_3} (2j_3+1)$



$=$

$= \sum_{j_3}$



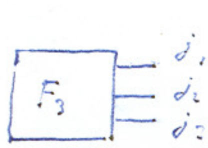
$(2j_3+1)$

$=$

$\delta_{j,0}\ \delta_{m,0}$

$= \frac{1}{2j_1+1}$

$\times$



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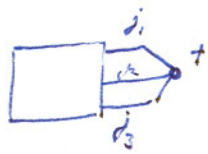
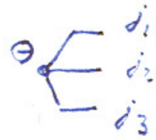
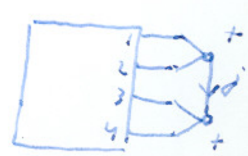


$\times$

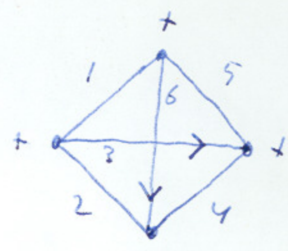


$= \sum_{j} (2j+1)$



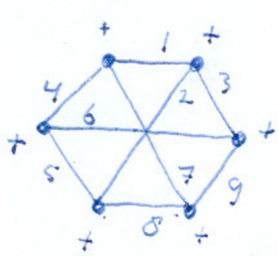
$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix} =$$

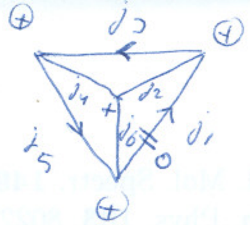
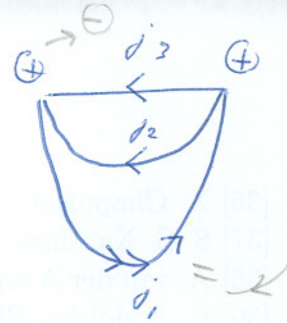
$=$



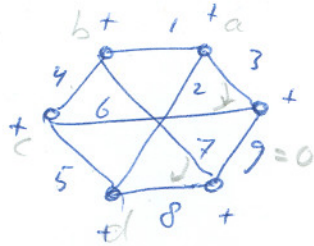
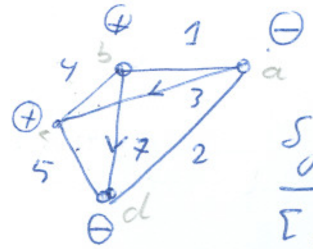
$(-1)^{-j_1+j_2+2j_3+j_4+j_5}$

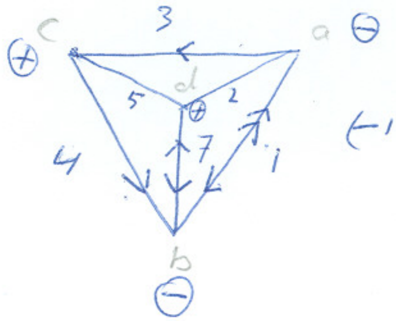
$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & j_9 \end{Bmatrix} =$$



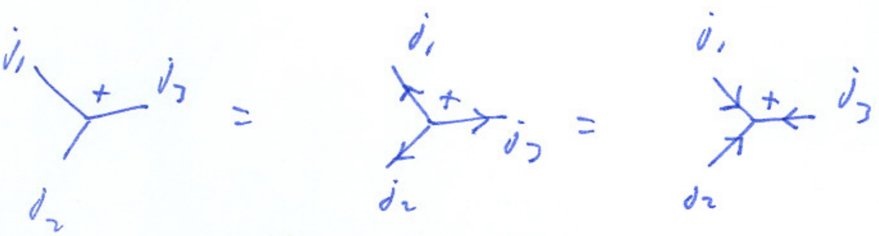
$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & 0 \end{Bmatrix} =$$  $$= \frac{\delta_{j_2 j_4}}{[j_2]} \frac{\delta_{j_1 j_5}}{[j_1]}$$ 

$$= \frac{\delta_{j_2 j_4} \delta_{j_1 j_5}}{[j_2]\,[j_1]} (-1)^{j_1+j_2+j_3} \Delta(j_1 j_2 j_3)$$

$$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \\ j_7 & j_8 & 0 \end{Bmatrix} =$$  $=$  $$\frac{\delta_{j_3 j_6}}{[j_3]} \quad \frac{\delta_{j_7 j_8}}{[j_7]}$$

$=$  $$(-1)^{2j_1 + j_1 + j_4 + j_7 + j_1 + j_2 + j_3}$$

$$= \frac{\delta_{j_3 j_6}}{[j_3]} \quad \frac{\delta_{j_7 j_8}}{[j_7]} \quad (-1)^{j_2+j_3+j_4+j_7} \quad \begin{Bmatrix} j_1 & j_2 & j_3 \\ j_5 & j_4 & j_7 \end{Bmatrix}$$



(feb 2004)

$$\bar{T}_{kq}(j_1, j_2) = \sum_{m_1 m_2} |j_1 m_1\rangle\langle j_2 m_2| \underbrace{(-1)^{j_1 - m_1} [k]^{1/2} \begin{pmatrix} j_1 & k & j_2 \\ -m_1 & q & m_2 \end{pmatrix}}$$

$[k]^{1/2}$