At Large distance angular momentum is quantized on $\underline{k}$:

25 may 2013

(1)

$$|j\Omega\rangle^{KF} \equiv \sum_{m_j} Y_{j m_j}(\theta_q, \phi_q)\, D^{j}_{m_j,\Omega}(\phi_k, \theta_k, 0)$$

↑ electron SF coordinates

$$|j\Omega\rangle^{KF} e^{i\underline{k}\cdot\underline{r}}$$

See Phys. Rev. A, 44, 347 (1991) Eq. (2.2

$$= \sum_{m_j} Y_{j, m_j}(\theta_q, \phi_q)\, D^{j}_{m_j, \Omega}(\phi_k, \theta_k, 0)\, 4\pi \sum_{l, m_l} i^{l} j_l(kr)\, Y_{l, m_l}(\theta_r, \phi_r)\, Y^{*}_{l, m_l}(\theta_k, \phi_k)$$

$$\Big[\; Y_{j, m_j}(\theta_q, \phi_q)\, Y_{l, m_l}(\theta_r, \phi_r) = \sum_{J M_J} \langle \hat{q}, \hat{r} (jl) J M_J\rangle \langle j m_j\, l m_l | J M_J\rangle$$

$$= \sum_{k} Y_{j,k}(\theta^{BF}, \phi^{BF})\, D^{J*}_{M_J, k}(\phi_r, \theta_r, 0) \sqrt{\frac{2J+1}{4\pi}}\, \mathcal{U}^{Jj}_{l,k} \langle j m_j\, l m_l | J M_J\rangle \;\Big]$$

$$= \sum_{\substack{m_j l m_l k \\ J M_J}} 4\pi\, i^{l} j_l(kr)\, Y_{j,k}(\theta^{BF}, \phi^{BF})\, D^{J,*}_{M_J, k}(\phi_r, \theta_r, 0) \sqrt{\frac{2J+1}{4\pi}}$$

$$\mathcal{U}^{Jj}_{l,k} \underbrace{\langle j m_j\, l m_l | J M_J\rangle\, D^{j}_{m_j, \Omega}(\phi_k, \theta_k, 0)\, Y^{*}_{l, m_l}(\theta_k, \phi_k)}$$

$$\sqrt{\frac{2l+1}{4\pi}}\, D^{J}_{M_J, \Omega}(\phi_k, \theta_k, 0) \langle j\Omega\, l 0 | J\Omega\rangle$$

$$= \sqrt{\frac{2J+1}{4\pi}}\, D^{J}_{M_J, \Omega}(\phi_k, \theta_k, 0)\quad \mathcal{U}^{J,j}_{l,\Omega}$$

$$= \sum_{J M_J} \sum_{l} \sum_{k} i^{l} j_l(kr)\, Y_{j,k}(\theta^{BF}, \phi^{BF})\, D^{J,*}_{M_J, k}(\phi_k, \theta_k, 0)(2J+1)$$

$$D^{J}_{M_J, \Omega}(\phi_k, \theta_k, 0)\quad \mathcal{U}^{Jj}_{l,k}\, \mathcal{U}^{Jj}_{l,\Omega}$$

$|j\ell\rangle^{KR} e^{i\mathbf{k}\cdot\mathbf{r}}$ (25 may 2013) (2)

$$= \sum_{J M_J, \lambda} (2J+1)\, Y_{j,\lambda}(\theta^{BF}, \phi^{BF})\, D^{J,*}_{M_J,\lambda}(\phi_k, \theta_k, 0)\, D^{J}_{M_J,\Omega}(\phi_f, \theta_f, 0)$$

$$\sum_\ell i^\ell j_\ell(kr)\, \mathcal{U}^{J,j}_{\ell,\lambda}\, \mathcal{U}^{J,j}_{\ell,\Omega}$$

$$j_\ell(kr) \cong \frac{\sin(kr - \frac{1}{2}\ell\pi)}{kr}$$

$$= \frac{e^{i(kr - \frac{1}{2}\ell\pi)} - e^{-i(kr - \frac{\ell\pi}{2})}}{2ikr}$$

$$= \frac{e^{ikr} i^{-\ell} - e^{-ikr} i^{\ell}}{2ikr}$$

$$i^\ell j_\ell(kr) \cong \frac{e^{ikr} - (-1)^\ell e^{-ikr}}{2ikr}$$

$$= \frac{e^{ikr}}{2ikr} + \frac{e^{-ikr}}{2ikr}(-1)^\ell$$

$$\sum_\ell \mathcal{U}^{J,j}_{\ell,\lambda}\, \mathcal{U}^{J,j}_{\ell,\Omega} = \delta_{\lambda,\Omega}$$

$$\sum_\ell (-1)^\ell\, \mathcal{U}^{J,j}_{\ell,\lambda}\, \mathcal{U}^{J,j}_{\ell,\Omega} =$$

$$= \sum_\ell (-1)^\ell \langle j\lambda\, \ell 0 | J\lambda\rangle \langle j\Omega\, \ell 0 | J\Omega\rangle \frac{2\ell+1}{2J+1}$$

$$\left[\langle j\lambda\, \ell 0 | J\lambda\rangle = (-1)^{j+\ell-J} \langle j, -\lambda\, \ell 0 | J, -\lambda\rangle \right]$$

25 may 2017

3

$$= \sum_{\ell} (-1)^{j-J} \langle j, -\lambda, \ell, 0 | J, -\lambda \rangle \langle j \Omega \ell 0 | J \Omega \rangle \frac{2\ell+1}{2J+1}$$

$$= (-1)^{j-J} \sum_{\ell} U^{J,j}_{\ell,-\lambda} U^{J,j}_{\ell,\Omega} = (-1)^{j-J} \delta_{\lambda,\Omega}$$

$$\sum_{\ell} i^{\ell} j_{\ell}(kr) U^{J,j}_{\ell,\lambda} U^{J,j}_{\ell,\Omega}$$

$$\simeq \frac{e^{ikr}}{2ikr} \delta_{\lambda,\Omega} + \frac{e^{-ikr}}{2ikr} \delta_{\lambda,-\Omega} (-1)^{j-J}$$

$$|j\Omega\rangle^{BF} e^{i\underline{k}\cdot\underline{r}} = \sum_{J,M_J} (2J+1)$$

$$\Big\{ Y_{j,\Omega}(\theta^{BF}, \phi^{BF}) D^{J*}_{M_J,\Omega}(\phi_k \theta_k 0) D^{J}_{M_J,\Omega}(\phi_k \theta_k 0) \frac{e^{ikr}}{2ikr}$$

outgoing

$$- Y_{j,-\Omega}(\theta^{BF}, \phi^{BF}) D^{J*}_{M_J,-\Omega}(\phi_k \theta_k 0) D^{J}_{M_J,-\Omega}(\phi_k \theta_k 0) \frac{e^{-ikr}}{2ikr} \Big\}$$

incoming $\times (-1)^{j-J}$

$$D^{J*}_{M_J,-\Omega} D^{J}_{M_J,-\Omega} =$$